Distributed coordination of spectrum and the prisoner's dilemma

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Abstract— **In this paper we discuss distributed spectrum allocation techniques in interference limited environment. We analyze the well known iterative water-filling strategy for a simplified two players interference game, and provide closed form analysis of the cases in which the IWF is suboptimal due to the occurrence of the well known prisoner's dilemma. We then propose some alternative distributed coordination algorithm and show its superiority over IWF for real wireline access network.**

I. Introduction

Distributed power and spectrum allocation is a fundamental problem for optimizing the use of a shared medium, by users that cannot apply a joint signaling scheme. The problem of distributed power control has been studied in the context of code division multiple access (CDMA) communication, while distributed spectrum allocation of wideband signals is much harder problem, due to the many degrees of freedom. Much work on distributed spectrum control has been done in the context of wireline networks. One of the most popular solutions to the problem of distributed spectrum allocation is the iterative water-filling (IWF) algorithm [8]. It is well known that the fixed points of the iterative water-filling algorithm are Nash equilibrium points of the Gaussian interference game [8]. However much less is known on the global optimality properties of these solutions. In this paper we concentrate on a simple two players version of the Gaussian interference game, and provide conditions under which the IWF solution is suboptimal due to the prisoner's dilemma phenomena, where the stable equilibrium point is bad for both users. The structure of the paper is as follows: In section II we review the Gaussian interference game. In section III we define the special two person version of the interference game. We define the cooperative frequency domain multiplexing (FDM) strategies and prove the division of the conditions under which the prisoner's dilemma occurs. The Near-Far problem in the digital subscriber line (DSL) environment as well as a simple distributed cooperative algorithm that solves this problem are introduced in IV. Finally in section V we provide some simulated experiments demonstrating the results. In an appendix we sketch an alternative existence proof of Nash equilibrium in the Gaussian interference game. We end up with some concluding remarks regarding possible extensions.

II. The Gaussian interference game

In this section we define the Gaussian interference game, and provide some simplifications for dealing with discrete frequencies. The Gaussian interference game was defined in [8]. In this section we define the discrete approximation of this game. Let $f_0 < \cdots < f_K$ be an increasing sequence of frequencies. Let I_k be the closed interval be given by $I_k = [f_{k-1}, f_k]$. We now define the approximate Gaussian interference game denoted by $GI_{\{I_1,\ldots,I_K\}}$.

Let the players $1,\ldots,N$ operate over K channels. Assume that the K channels have crosstalk coupling functions $h_{ij}(k)$. Assume that user i'th is allowed to transmit a total power of P_i. Each player can transmit a power vector $\mathbf{p}_i = (p_i(1),\ldots,p_i(K)) \in [0,P_i]^K$ such that $p_i(k)$ is the power transmitted in the interval I_k. Therefore we have $\sum_{k=1}^K p_i(k) = P_i$. The equality follows from the fact that in non-cooperative scenario all users will use the maximal power they can use. This implies that the set of power distributions for all users is a closed convex subset of the cube $\prod_{i=1}^N [0,P_i]^K$ given by:

$$\mathbf{B} = \prod_{i=1}^N \mathbf{B}_i \tag{1}$$

where $\mathbf{B}_i$ is the set of admissible power distributions for player i is

$$\mathbf{B}_i = [0,P_i]^K \cap \left\{ (p(1),\ldots,p(K)) : \sum_{k=1}^K p(k) = P_i \right\} \tag{2}$$

Each player chooses a power spectral density (PSD) $\mathbf{p}_i = \langle p_i(k) : 1 \le k \le K \rangle \in \mathbf{B}_i$. Let the payoff for user i be given by:

$$C^i(\mathbf{p}_1,\ldots,\mathbf{p}_N) = \sum_{k=1}^K \log_2\left(1 + \frac{|h_i(k)|^2 p_i(k)}{\sum |h_{ij}(k)|^2 p_j(k) + \mathbf{n}(k)}\right) \tag{3}$$

where C^i is the capacity available to player i given power distributions $\mathbf{p}_1,\ldots,\mathbf{p}_N$, channel responses $h_i(f)$, crosstalk coupling functions $h_{ij}(k)$ and $n_i(k) > 0$ is external noise present at the i'th channel receiver at frequency k. In cases where $n_i(k) = 0$ capacities might become infinite using FDM strategies, however this is non-physical situation due to the receiver noise that is always present, even if small. Each C^i is continuous on all variables.

Definition 1: The Gaussian Interference game $GI_{\{I_1,\ldots,I_k\}} = \{\mathbf{C},\mathbf{B}\}$ is the N players non-cooperative game with payoff vector $\mathbf{C} = \left(C^1,\ldots,C^N\right)$ where C^i are defined in (3) and $\mathbf{B}$ is the strategy set defined by (1).

The interference game is an N players game, that captures the process of independent users trying to optimize their own data rate under power constraint and interference from other users. One well known strategy for distributively optimizing

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the spectrum is based on each user independently (and iteratively) optimizing its own rate by choosing a spectrum (under power constraint) against the other users interference and the background noise. This process is known as iterative waterfilling (IWF) since for each user the optimal way to allocate its own rate, given the power spectral density of all other users is to distribute the power such that the sum of the total interference and noise and its own power is constant [10]. Convergence of the iterative process is assured in many situations into unique spectrum allocation for the various users [2]. Due to its local optimality the IWF algorithm gained popularity for optimizing the spectrum in multiple access channels for the single antenna as well as the vector case [8]. Interestingly the fixed point of the IWF algorithm is a Nash equilibrium point in the interference game. It can be shown that at least one Nash equilibrium point exists [1], and under certain conditions this Nash equilibrium point is unique [2] (See appendix for a sketch of an alternative proof that provides more insight into the water-filling strategies). However as is well known, Nash equilibrium might be highly suboptimal for both players. This situation is known in the game theoretic literature as the prisoner's dilemma [6].

In this paper we will analyze a simple version of the two players interference game and provide certain conditions under which the prisoner's dilemma occurs when using the IWF algorithm.

III. The Prisoner's Dilemma

The IWF algorithm maximizes each user's rate (by allocating the power through waterfilling) without taking into account the influence of such an allocation on the other user's rate. This strategy can be viewed as a very pragmatic one since cooperation is not possible between the users, but a-priori agreement regarding choice of strategies can be used. We will show here that even without cooperation a better scheme of allocating the power can be adopted in some cases, even for very simple interference channels. To that end we introduce a special case of the two players interference game. For this case it is possible to determine analytically the channels for which IWF is optimal and the set of channels where IWF leads to prisoner's dilemma, a situation discovered by Flood and Dresher [7]. Furthermore we will also show a third case where *two* Nash equilibrium points exist, and the IWF equilibrium has lower sum-rate than the other Nash equilibrium. In this special version of the interference game we assume that the two users share two independent frequency bands, with symmetric and identical channel conditions for each band.

We can represent the channel as follows:

$$|H(1)|^2 = |H(2)|^2 = \begin{bmatrix} 1 & h \\ h & 1 \end{bmatrix} \tag{4}$$

where:
$H(1)$ and $H(2)$ are the normalized channel matrices for each frequency band, and
$h = |h_{12}(1)|^2 = |h_{21}(1)|^2 = |h_{12}(2)|^2 = |h_{21}(2)|^2$
We also assume that both users have the same power constraint P and the power allocation matrix is defined as

$$P \cdot \begin{bmatrix} 1-\alpha & \alpha \\ \beta & 1-\beta \end{bmatrix} \tag{5}$$

where $0 \le \alpha, \beta \le 1$.
The capacity for user I is given by:

$$C^1 = \frac{1}{2}\log_2\left(1 + \frac{(1-\alpha)\cdot P}{N + \beta \cdot P \cdot h}\right) + \frac{1}{2}\log_2\left(1 + \frac{\alpha \cdot P}{N + (1-\beta)\cdot P \cdot h}\right) \tag{6}$$

where N is the noise PSD. Simplifying (6) we obtain that user I's payoff (capacity) is:

$$C^1 = \frac{1}{2}\log_2\left(1 + \frac{1-\alpha}{SNR^{-1} + \beta \cdot h}\right) + \frac{1}{2}\log_2\left(1 + \frac{\alpha}{SNR^{-1} + (1-\beta)\cdot h}\right) \tag{7}$$

Similarly the capacity of user II is given by:

$$C^2 = \frac{1}{2}\log_2\left(1 + \frac{\beta}{SNR^{-1} + (1-\alpha)\cdot h}\right) + \frac{1}{2}\log_2\left(1 + \frac{1-\beta}{SNR^{-1} + \alpha \cdot h}\right) \tag{8}$$

We would like to interpret cooperation as choosing to use only one of the two bands, providing the other player an interference free band, while competition will be interpreted as waterfilling over the two bands. α and β can be interpreted as the level of mutual cooperation (α determines the level of cooperation of user I with II, and β the level of cooperation of user II with I). The cooperative strategy (FDM) is causing no interference to the other user (by allocating all the power at one band), this implies that $\alpha = \beta = 0$. A Competitive act will be to maximize the capacity using water filling strategy.

We are now ready to analyze the impact of competitive strategies on the two users payoffs (capacities). According to water filling strategy, (in the case of equal power constraint and white noise PSD) for given β we need to choose α such that the following equation holds:

$$(1-\beta)h + \alpha = \beta h + (1-\alpha) \tag{9}$$

which implies that

$$\alpha = \frac{(2\beta - 1)h + 1}{2} \tag{10}$$

since the channel is symmetric β is given by

$$\beta = \frac{(2\alpha - 1)h + 1}{2} \tag{11}$$

Therefore in the symmetric case the IWF will converge to

$$\alpha = \beta = \frac{1}{2} \tag{12}$$

Table I and II summarize the payoffs of users I,II in our simplified game respectively at four different levels of mutual cooperation.

TABLE I

USER I PAYOFFS AT DIFFERENT LEVELS OF MUTUAL COOPERATION

	user II is fully cooperative $(\beta = 0)$	user II is fully competing $\left(\beta = \frac{(2\alpha-1)h+1}{2}\right)$
user I is fully cooperative $(\alpha = 0)$	$\frac{1}{2}\log_2\left(1+\frac{1}{SNR^{-1}}\right)$	$\frac{1}{2}\log_2\left(1+\frac{1}{SNR^{-1}+\frac{(1-h)}{2}h}\right)$
user I is fully competing $\left(\alpha = \frac{(2\beta-1)h+1}{2}\right)$	$\frac{1}{2}\log_2\left(1+\frac{\frac{1+h}{2}}{SNR^{-1}}\right)+\frac{1}{2}\log_2\left(1+\frac{\frac{1-h}{2}}{SNR^{-1}+h}\right)$	$\log_2\left(1+\frac{\frac{1}{2}}{SNR^{-1}+\frac{1}{2}h}\right)$

TABLE II

USER II PAYOFFS AT DIFFERENT LEVELS OF MUTUAL COOPERATION

	user II is fully cooperative $(\beta = 0)$	user II is fully competing $\left(\beta = \frac{(2\alpha-1)h+1}{2}\right)$
user I is fully cooperative $(\alpha = 0)$	$\frac{1}{2}\log_2\left(1+\frac{1}{SNR^{-1}}\right)$	$\frac{1}{2}\log_2\left(1+\frac{\frac{1+h}{2}}{SNR^{-1}}\right)+\frac{1}{2}\log_2\left(1+\frac{\frac{1-h}{2}}{SNR^{-1}+h}\right)$
user I is fully competing $\left(\alpha = \frac{(2\beta-1)h+1}{2}\right)$	$\frac{1}{2}\log_2\left(1+\frac{1}{SNR^{-1}+\frac{(1-h)}{2}h}\right)$	$\log_2\left(1+\frac{\frac{1}{2}}{SNR^{-1}+\frac{1}{2}h}\right)$

A prisoner's dilemma situation is defined by the following payoff relation - $T > R > P > N$ as well as $2R > T + N$ [6], where

- T (Temptation) is one's payoff for defecting (choosing a competitive strategy) while the other cooperates.
- R (Reward) is the payoff of each player where both cooperate.
- P (Penalty) is the payoff of each player where both defect.
- N (Naive) is one's payoff for cooperating while the other defects.

It is easy to show that the Nash equilibrium point in this case is that both players will defect (P). This is caused by the fact that given the other user's act the best response will be to defect (since $T > R$ and $P > N$). Obviously a better strategy (which makes this game so interesting) is mutual cooperation (since $R > P$).

In our context, the IW algorithm obtains the mutually competitive payoff (2P), since each player maximizes its own data rate while others can be severely harmed by its allocation. For the cooperative action we choose in this paper the FDM like algorithms which are cooperative since each user maximizes its own rate without harming the other user's data rates.

As before mentioned a prisoner's dilemma situation is characterized by the following payoff relations: $T > R > P > N$, with additional condition on the sum-rate: $2R > T + N$. The latter condition implies that a mixed strategy (i.e. one user is cooperating while the other competing) will not achieve higher sum rate than mutual cooperation. By examining the relations between the different rates (payoffs) we can derive a set of conditions on h and SNR for which the given interference game with the set of action that can be carried out by the users (IWF and FDM) defines a prisoner's dilemma situation:

(a) $T > R$:

$$\frac{1}{2}\log_2\left(1+\frac{\frac{1+h}{2}}{SNR^{-1}}\right)+\frac{1}{2}\log_2\left(1+\frac{\frac{1-h}{2}}{SNR^{-1}+h}\right)$$

$$> \frac{1}{2}\log_2\left(1+\frac{1}{SNR^{-1}}\right) \tag{13}$$

this equation reduced to $h^2 - 2 \cdot h + 1 > 0$ which holds for every $h \neq 1$.

(b) $T > P$:

$$\frac{1}{2}\log_2\left(1+\frac{\frac{1+h}{2}}{SNR^{-1}}\right)+\frac{1}{2}\log_2\left(1+\frac{\frac{1-h}{2}}{SNR^{-1}+h}\right)$$

$$> \log_2\left(1+\frac{\frac{1}{2}}{SNR^{-1}+\frac{1}{2}h}\right) \tag{14}$$

simplifying the equation we obtain

$$SNR^{-2}\left(h+\frac{1}{4}h^2\right)+SNR^{-1}\left(\frac{1}{2}h^3+\frac{3}{4}h^2+\frac{1}{4}h\right)$$

$$+\left(\frac{1}{16}h^4+\frac{1}{8}h^3+\frac{1}{16}h^2\right) > 0 \tag{15}$$

since SNR and h are nonnegative the equation always true.

(c) $R > P$

$$\frac{1}{2}\log_2\left(1+\frac{1}{SNR^{-1}}\right) > log_2\left(1+\frac{\frac{1}{2}}{SNR^{-1}+\frac{1}{2}h}\right) \tag{16}$$

simplifying (15) we get

$$h^2 + 2hSNR^{-1} - SNR^{-1} > 0 \tag{17}$$

since h is nonnegative the equation holds for $h > h_{\lim 1}$, where

$$h_{\lim 1} = SNR^{-1}\left(\sqrt{1+\frac{1}{SNR^{-1}}}-1\right) \tag{18}$$

(d) $R > N$

$$\frac{1}{2}\log_2\left(1+\frac{1}{SNR^{-1}}\right) > \frac{1}{2}\log_2\left(1+\frac{1}{SNR^{-1}+\frac{(1-h)}{2}h}\right) \tag{19}$$

which reduced to $\frac{1-h}{2} \cdot h > 0$, this equation holds for every $0 \leq h < 1$.

(e) $P > N$

$$\log_2\left(1 + \frac{\frac{1}{2}}{SNR^{-1} + \frac{1}{2}h}\right) > \frac{1}{2}\log_2\left(1 + \frac{1}{SNR^{-1} + \frac{(1-h)}{2}h}\right) \quad (20)$$

or equivalently

$$h^3 + h^2(0.5 + 2SNR^{-1}) - 0.5h - SNR^{-1} < 0 \quad (21)$$

since h is nonnegative the equation holds for $h < h_{\lim 2}$, where $h_{\lim 2}$ is the solution for (20) given by the cubic formula.
finally, since the maximization is done on the sum rate we have to check whether -

(f) $2R > T + N$:

$$\log_2\left(1 + \frac{1}{SNR^{-1}}\right) > \frac{1}{2}\log_2\left(1 + \frac{\frac{1+h}{2}}{SNR^{-1}}\right) + \frac{1}{2}\log_2\left(1 + \frac{\frac{1-h}{2}}{SNR^{-1} + h}\right) + \frac{1}{2}\log_2\left(1 + \frac{1}{SNR^{-1} + \frac{(1-h)}{2}h}\right) \quad (22)$$

which reduced to

$$SNR^{-2}\left(6(1-h^2) + 8h\right) + SNR^{-1}(9h + h^2) + 4h^2(1-h) > 0 \quad (23)$$

since h and SNR are nonnegative the equation is true in the relevant region of $0 \leq h < 1$ for every SNR.

Combining all the relation above we conclude that only three situation are possible:

- (A) $T > P > R > N$, for $h < h_{\lim_1}$
- (B) $T > R > P > N$, for $h_{\lim_1} < h < h_{\lim_2}$
- (C) $T > R > N > P$, for $h_{\lim_2} < h$

where $h_{\lim_1}$ and $h_{\lim_2}$ are given above.
The sum rate is either $2 \cdot R$ (for mutual cooperation), $2 \cdot P$ (for mutual competition) or $T + N$ (for mixed strategy). Examining the achieved sum rate for the two strategies (IWF and FDM) yields the following:

The payoffs relations in (A) correspond to a game called "Deadlock". In this game there is no dilemma, since as in the prisoner's dilemma situation, no matter what the other player does, it is better to defect ($T > R$ and $P > N$), so the Nash equilibrium point is P. In contrast to prisoner's dilemma, in this game $P > R$ thus there is no reason to cooperate. The maximum sum rate is also $2P$ because $2 \cdot R > T + N$ and $P > R$. Since applying the IWF strategy equals to P (by our definition of competition), this is the region where the IWF algorithm achieves the maximum sum rate.

The payoffs relations in (B) corresponds to the above discussed prisoner's dilemma situation. While the Nash equilibrium point is P, the maximum sum rate is achieved by R. In this region the FDM strategy will achieve the maximum sum rate.

The last payoffs relations (C) corresponds to a game called "Chicken". This game is characterized by having two distinguished Nash equilibrium points, T and N. This is caused by the fact that for each player's strategy a *different* response is better (if the other cooperates it is better to defect since $T > R$, while if the other defects it is better to cooperate since $N > P$). The maximum rate sum point is still at R (since $R > P$ and $2 \cdot R > T + N$) thus, again FDM will achieve the maximum rate sum while IWF won't. The three different regions as a function of SNR and channel coefficient h are depicted in figure 1

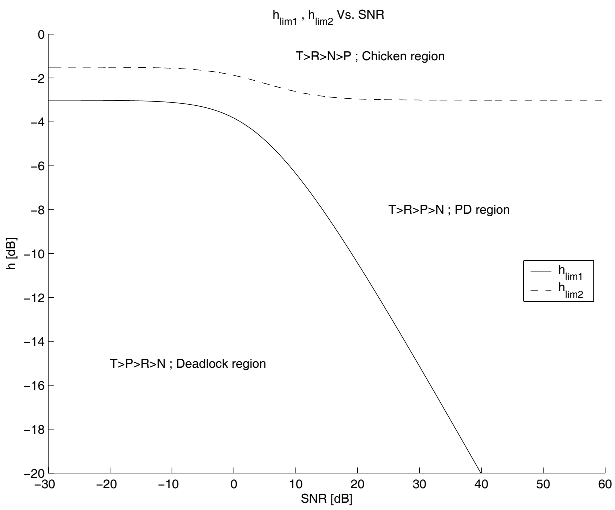

Fig. 1. $h_{\lim_1}$ and $h_{\lim_2}$ vs. SNR, The solid line corresponds to $h_{\lim_1}$ and the dashed line corresponds to $h_{\lim_1}$

IV. The Near-Far Problem in DSL

One of the most acute problem in the DSL environment is having loops with different length sharing the same binder. This problem is common both in the upstream where different users with different loop length share the same central office (CO) and in the downstream where a CO shares the same binder with a remote terminal (RT) located closer to the destination. The insertion loss (or the main channel path gain) of a DSL loop have the characteristic of low pass filter while the crosstalk coefficients have high pass nature. This behavior of the DSL lines causes severe problem when having one or more loops longer than others as it will be demonstrate in the following.

Consider a general 2×2 channel matrix where both loops have the same length L, the bands width are normalize to 1 and the power constraint for user I and II are P_1 and P_2 respectively. The channel matrices are of the form

$$|H_L(k)|^2 = \begin{bmatrix} h_1(k) & h_{12}(k) \\ h_{21}(k) & h_2(k) \end{bmatrix} \quad (24)$$

where $1 \leq k \leq K$

We use this basic channel to construct more complex topology where the loops doesn't share the same length. Consider the general topology in Fig. 2, in this topology there is a far user with a total length of $l_1 \cdot L$, near user with a total length of $l_2 \cdot L$ and the overlap length is L (the terms far and near correspond to the length from the loops overlap section). This channel is represented by the following matrix

$$|H(k)|^2 =$$
$$\begin{bmatrix} 1 & 0 \\ 0 & h_2^{l_2-1}(k) \end{bmatrix} \cdot \begin{bmatrix} h_1(k) & h_{12}(k) \\ h_{21}(k) & h_2(k) \end{bmatrix} \cdot \begin{bmatrix} h_1^{l_1-1}(k) & 0 \\ 0 & 1 \end{bmatrix} \tag{25}$$

resulting in

$$|H(k)|^2 = \begin{bmatrix} h_1^{l_1}(k) & h_{12}(k) \\ h_2^{l_2-1}(k)h_{21}(k)h_1^{l_1-1}(k) & h_2^{l_2}(k) \end{bmatrix} \tag{26}$$

It is easy to see from the channel matrix that user I's (the far one) attenuation is severe (in comparison to the basic loop attenuation) while his crosstalk coefficient remains the same as for the basic loop. Combined with the above mentioned nature of the DSL loops this scenario causes the far user to transmit only at the low frequency region where he obtains positive signal to noise ratio even in the presence of the far user's interference. Figure 3 depicts the channel transfer function and crosstalk coefficient for such a topology where L =0.9 km , $l_1 = 4$ and $l_2 = 2$.

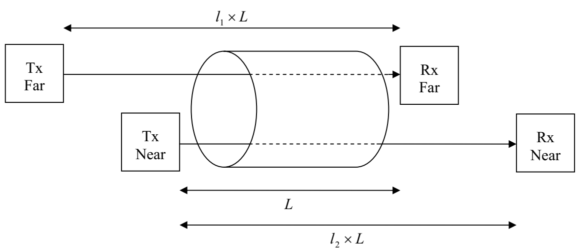

Fig. 2. Loop topology of the Near-Far problem in DSL.

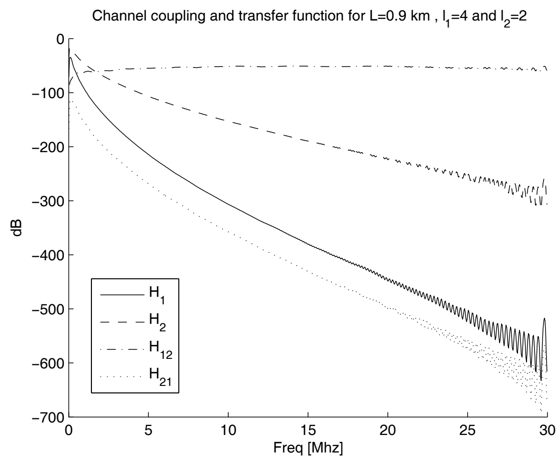

Fig. 3. Main channels and crosstalk gain for 2×2 near far channel.

Obviously, user II (the near one) dominates the channel since his crosstalk into the other line is large while the crosstalk of the far user is negligible. Applying a rate adaptive IWF (RA-IWF) in this case will influence badly on the far user rate. This caused by the fact that RA-IWF will result in using all the power available for the near user and the use of low frequencies region (which have the lowest noise PSD profile). The combination of using the low frequencies region with full power available causes the maximum interference to the far user resulting in poor achievable rate.

The "competitive" solution for such a scenario is the Fixed Margin IWF (FM-IWF). In this form of IWF there is a design rate R_d for the near user and we adjust the near user rate by applying power backoff on the total power constraint. This power backoff procedure mitigate the interference caused to the far user.

A. *The dynamic FDM algorithm*

Inspired by the benefits of cooperative strategies and further analysis that will be presented elsewhere [16] we propose the cooperative solution for the near-far problem. The dynamic FDM (DFDM) algorithm, first presented in [15], allocates the power of the near user not only as a reaction of its noise PSD (as the IWF does) but by minimizing the use of the lower part of the spectrum. Since the far user can allocate its power only at the lower part of spectrum, applying the DFDM on the far user power allocation reduces the level of interference to the far user by means of orthogonal transmitting bands. The idea underlying the approach above is that the far user uses the lower part of the spectrum (as explained above), and therefore use of this part of the spectrum should be minimized for the near user.

We define f_c to be the cutoff frequency i.e. the minimal frequency used by the near user. The power allocation method in the DFDM algorithm is as following - given R_d the design rate of the near user, the near user allocates its power such that the rate achieved is equal to R_d along with maximizing f_c. Actually the algorithm is implemented in two steps, at the first one the maximal f_c is found (this step is performed by applying RA-IWF at varying f_c values). The second step is reducing the total power by applying FM-IWF on the upper part of the spectrum determined by the former step. The implementation steps of the DFDM algorithm are summarized in table III.

V. Simulation

In this section we provide some simulated experiments demonstrating the rate region under various strategies. We provide examples of the scenarios discussed above: The deadlock and the prisoner's dilemma. Figures 2 and 3 shows the rate of user I and the sum rate respectively for channel with $SNR = 20[dB]$ (for which $h_{\lim_1} = 0.09$) and $h = 0.2$. Since $h > h_{\lim_1}$ FDM ($\alpha = \beta = 0$) sum rate is higher than IWF ($\alpha = \beta = 0.5$) sum rate.

Figures 4 and 5 shows the rate of user I and the sum rate respectively for channel with the same SNR as before ($SNR = 20[dB]$) and $h = 0.01$. Since $h < h_{\lim_1}$ IWF ($\alpha = \beta = 0$) sum rate is higher than FDM ($\alpha = \beta = 0.5$) sum rate.

Finally we demonstrate results of the DFDM algorithm in comparison to FM-IWF for wireline communications. We present the rate region for two groups of modems. 8 located

TABLE III
DFDM IMPLEMENTATION FOR THE NEAR-FAR SCENARIO

1. Let R_d = preassigned target rate for the near user.
2. find f_c, the minimal f such that the near user can achieve rate R_d using frequencies above f_c.
3. Allocate the minimal amount of power needed for achieving R_d using only frequencies grater than f_c.

at a remote terminal located 900m from the customer, sharing a binder with 8 CO based ADSL modems at distance of 4.5 km. We can clearly see the better rate region of cooperative DFDM algorithm, compared to the FM-IWF algorithm.

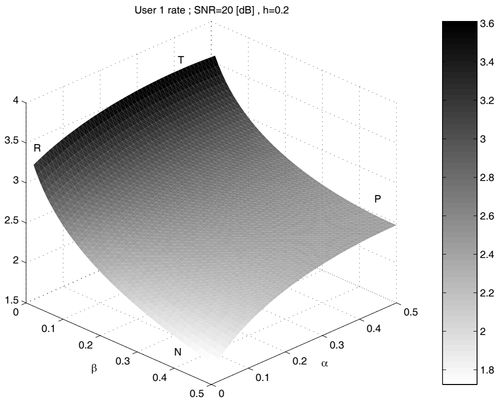

Fig. 4. Graph of user I payoff for different levels of cooperation ($0 \leq \alpha, \beta \leq 0.5$), since $h = 0.2 > h_{lim} = 0.09$ @$SNR = 20$ [dB] a prisoner's dilemma holds for this game (i.e. $T > R > P > N$).

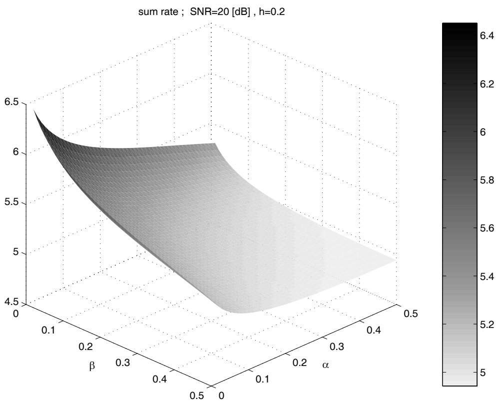

Fig. 5. The sum rate of the two users Gaussian interference game for different levels of cooperation ($0 \leq \alpha, \beta \leq 0.5$); mutual cooperation strategy (FDM ,$\alpha = \beta = 0$) maximizes the sum rate.

VI. CONCLUSIONS

In this paper we have discussed competitive and cooperative distributed spectrum coordination techniques for the two users Gaussian interference game. We have analyzed the cases where IWF strategies are subject to the prisoner's dilemma. We have also demonstrated the possible gain in cooperative spectrum allocation techniques on measured DSL channels. We note that the results are equally relevant for wireline and wireless channels. In an extension of this work [16] we define another

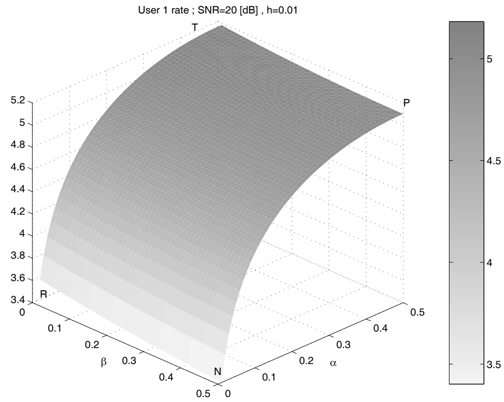

Fig. 6. Graph of user I payoff for different levels of cooperation ($0 \leq \alpha, \beta \leq 0.5$), since $h = 0.01 < h_{lim} = 0.09$ @$SNR = 20$ [dB] there is no dilemma (i.e. $T > P > R > N$).

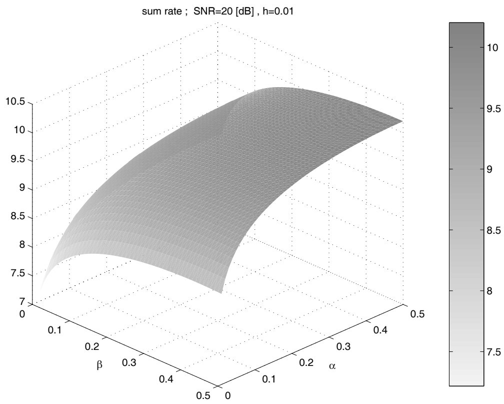

Fig. 7. The sum rate of the two users Gaussian interference game for different levels of cooperation ($0 \leq \alpha, \beta \leq 0.5$); IWF method ($\alpha = \beta = 0.5$) maximizes the sum rate.

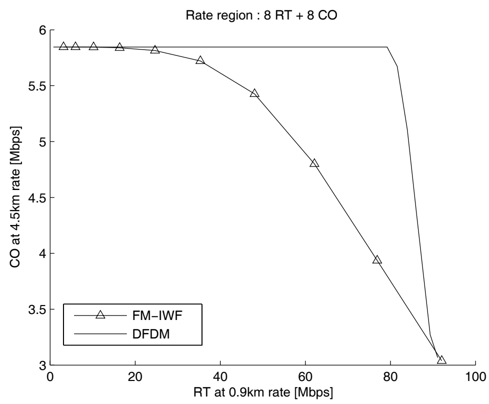

Fig. 8. Comparison of the rate region for 8 Remote terminals at 900m and 8 central office ADSL modems at L=4.5 using FM-IWF and DFDM.

type of game that describes near-far problems. This game is caled the "Bully game". We then provide tight bounds on rate region for FM-IWF and DFDM for the case of near-far interference channel.

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VII. APPENDIX

In this section we prove that for every sequence of intervals $\{I_1, \ldots, I_k\}$,the Gaussian interference game has a Nash equilibrium point. Our proof is based on the technique of [4], (see also [5]), adapted to the water-filling strategies in the GI game. While the result follows from standard game theoretic results, it is interesting to see the continuity of the water-filling strategy as the reason for the existence of the Nash equilibrium.

Theorem 1: For any finite partition $\{I_1, \ldots, I_k\}$ a Nash equilibrium in the Gaussian interference game $GI_{\{I_1,\ldots,I_k\}}$ exists.

Proof: For each player i define the water-filling function $W_i(\mathbf{p}_1, \ldots, \mathbf{p}_N) : \mathbf{B} \rightarrow \mathbf{B}_i$, which is the power distribution that maximizes C^i given that for every $j \neq i$ player j uses the power distribution $\mathbf{p}_j$ subject to the power limitation P_i. The value of $W_i(\mathbf{p}_1, \ldots, \mathbf{p}_n)$ is given by water-filling with total power of P_i against the noise power distribution composed of

$$N_i(k) = \frac{1}{|h_i(k)|^2}\left[\sum_{j\neq i} |h_ij(k)|^2 p_j(k) + n_i(k)\right] \tag{27}$$

where for all k, $n_i(k) > 0$ is the external noise power in the k'th band.

Claim 2: $W_i(\mathbf{x}_1, \ldots, \mathbf{x}_N)$ is a continuous function.

Proof: We shall not prove this in detail. However informally this fact is very intuitive since small variations in the noise and interference power distributions will lead to small changes in the waterfilling response.

The proof of theorem 1 now easily follows from the Brauwer fixed point theorem. The function $\mathbf{W} = [W_1, \ldots, W_N]$ maps $\mathbf{B}$ into itself. Since $\mathbf{B}$ is compact subset of a finite dimensional Euclidean space $\mathbf{W}$ has a fixed point $[\mathbf{p}_1, \ldots, \mathbf{p}_N]^T$. This means that

$$\mathbf{W}([\mathbf{p}_1, \ldots, \mathbf{p}_N]^T) = [\mathbf{p}_1, \ldots, \mathbf{p}_N]^T$$

By the definition of $\mathbf{W}$ this means that each $\mathbf{p}_i$ is the result of player i water-filling its power against the interference generated by $\{\mathbf{p}_j : j \neq i\}$ subject to its power constrain . Therefore $[\mathbf{p}_1, \ldots, \mathbf{p}_N]^T$ is a Nash equilibrium for $GI_{\{I_1,\ldots,I_K\}}$.