A low complexity linear precoding technique for next generation VDSL downstream transmission over copper

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Abstract

In this paper we study a simplified linear precoding scheme for far end crosstalk (FEXT) cancellation in Very High bit-rate DSL (VDSL) downstream transmission. We compare the proposed method to Zero-Forcing (ZF) FEXT cancellation and show that for multi-pair VDSL systems the method achieves rates that are close to the ZF precoding method. We also derive simple lower bounds on the performance that allows us to predict the performance of the proposed algorithm on each tone. We end up with testing the proposed method on theoretical and empirical channels. This is the first extensive study of VDSL precoding on measured channel data. The proposed method is less complex than ZF precoding and therefore alleviates the computational load at the initialization and tracking of precoded Very high bit-rate DSL (VDSL, VDSL2) systems. Finally we discuss implementation complexity in terms of total Silicon area and show that the method has favorable implementation complexity, since only channel coefficients need to be stored during steady-state transmission.

Key words: VDSL, VDSL2, digital subscriber line, vectored transmission, multichannel receivers, crosstalk mitigation, Multichannel transmission, linear precoding

EDICS: 3-comm, 3-TDSL, 3-ACCS, 3-CEQU

I. Introduction

The ever increasing demand for bandwidth in the access network has pushed VDSL technology to a point where a single modem (e.g. VDSL modem [1]) operating independently of other modems in the same binder almost achieves the single channel achievable rate where the achievable rate is limited by crosstalk from other users. While there are about 2-3 dB that can still be improved using advanced coding techniques such as low density parity check codes, these might improve the VDSL rate at a given distance by at most 1 bps/Hz.

It has been shown, e.g. [9], that if one coordinates the complete binder a substantial increase in the achievable rate of each user is attained. In [12], [13] we analyze the case of partial binder coordination and show that there is a need to process at least half of the operating pairs in order to obtain substantial gain in achievable rate. In a typical deployment, the fiber optic network is terminated at an optical network unit (ONU). Data is further distributed over the existing copper infrastructure to the various users. This topology eliminates the need for expensive optical transceivers for each end user and allows sufficient bandwidth to each user. Due to the distributed nature of the

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customer premises equipment (CPE) any joint transmission or reception must take place at the optical network termination (ONU). Recently several papers have dealt with full coordination of downstream VDSL transmission. Ginis and Cioffi [9] proposed a Tomlinson Harashima type non-linear FEXT precoding. Cendrillon et. al [3], [2], have noted that it is sufficient to use a linear precoding due to the diagonal dominance property of the FEXT coupling matrix, still their ZF solution requires matrix inversion at each tone of the multichannel DSL system. While ZF precoders have been widely used in the wireless communication see, e.g, [4], [5], the novel approximate precoding techniques proposed here are inappropriate for wireless communications due to the lack of (row) diagonal dominance in fading channels. The computational savings in the proposed method are important for initializations of existing systems. Furthermore the proposed method can lead to significant Silicon saving in the implementation of steady state transmission at the expense of slightly increased computational complexity. This is done by alleviating the need to keep the precoder coefficients and using directly the channels and the frequency domain equalizer coefficients of each modem.

For a standard VDSL system 4096 inversions of matrices of order of 25 are necessary (e.g., the quad binders used in this paper contain 28 twisted pairs). For next generation VDSL2 and VDSL3 we might have to use 8192 tones to utilize the full bandwidth up to 30 MHz.

In this paper we consider the possibility of achieving the FEXT precoding for downstream transmission using a suboptimal but computationally efficient scheme. We consider several approximations to the ZF linear precoder based on the row-wise diagonal dominance of the channel response matrix $\mathbf{H}$. For a large number of pairs our solution is substantially simpler with minor degradation in the achievable rates. It should be further pointed out that since both methods are based on precoding it is possible to select the correct scheme for each tone based on the channel matrix without substantial performance penalty using the performance bounds provided in this paper. Finally we compare the performance of the proposed method in terms of data rate to ZF FEXT cancellation and to no FEXT cancellation using measured crosstalk channels. This is the first experimental verification of such techniques. The experimental data has been provided by France Telecom labs. The full setup, as well as statistical characterization of the channels, are described in [11]. The extended simulations also cover the important case of mixed length loops where real loops of lengths between 75m and 600m are jointly precoded demonstrating the relevance of the method to real life scenarios and provide empirical cumulative distribution functions for various precoding techniques. This allows observing outages as well as ergodic capacities.

The structure of the paper is as follows: In section II we describe a mathematical model for multi-pair DSL systems. In section III we describe two simplified FEXT precoding schemes based on first and second order approximations. In section IV we analyze the performance of the proposed methods and provide some lower bounds on the achievable rates. In section V we describe simulated experiments using real channel measurements that verify the performance of the proposed methods. We end up with a detailed analysis of the implementation complexity of the proposed precoders VI and conclude with some remarks on the relevance of the proposed techniques.

II. Signal model

In this section we describe the signal model of a multichannel precoded system. We concentrate on discrete multi-tone (DMT) systems where the transmission is done independently over many narrow sub-bands. Assume that we have a binder consisting of p twisted pairs, typical binders include 25, 28, 50 or 100 pairs. A system coordinating the transmission of all p pairs numbered $1, \ldots, p$ is operating in the binder. We further assume that the systems under consideration operate in a frequency division duplexing mode (FDD), where upstream and downstream transmissions are performed at separate frequency bands, similar to VDSL, and that the systems in the binder are synchronized similar to the emerging VDSL2 standard. Hence near end crosstalk (NEXT) is eliminated. The received signal at all pairs i, $1 \leq i \leq p$ assuming a DMT modulation can be written in vector form as

$$\mathbf{x}(f_k) = \mathbf{H}(f_k)\mathbf{s}(f_k) + \boldsymbol{\nu}(f_k) \tag{1}$$

where

$$\mathbf{H}(f_k) = \begin{bmatrix} h_{11}(f_k) & \cdots & h_{1p}(f_k) \\ \vdots & \ddots & \vdots \\ h_{p1}(f_k) & \cdots & h_{pp}(f_k) \end{bmatrix}$$

is the channel frequency response, $\mathbf{s}(f_k) = [s_1(f_k), \ldots, s_p(f_k)]^T$ are the frequency domain representations of the signals transmitted by the system and $\nu(f_k)$ is a vector of additive white Gaussian noise (AWGN) typically assumed to have a PSD of -140 dBm/Hz. Since we assume a discrete multi-tone transmission we shall assume that f_k correspond to the frequency bins of the specific DMT system at hand, and the signals at each frequency are typically QAM modulated signals with modulation level determined by the SNR at the receiver at the given frequency. The modulation level varies from BPSK up to 2^{15} QAM when the signal to noise ratio is sufficiently good. When the specific frequency processed is not relevant for the discussion we shall suppress the explicit dependence on f_k and use the following notation

$$\mathbf{x} = \mathbf{H}\mathbf{s} + \boldsymbol{\nu} \tag{2}$$

A ZF linear precoder premultiplies $\mathbf{s}$ by $\mathbf{H}^{-1}\mathbf{D}$ where $\mathbf{D}$ is the diagonal matrix

$$\mathbf{D} = \begin{bmatrix} h_{11} & & \\ & \ddots & \\ & & h_{pp} \end{bmatrix} \tag{3}$$

The received signal now becomes

$$\mathbf{x} = \mathbf{D}\mathbf{s} + \boldsymbol{\nu} \tag{4}$$

and therefore we obtain a FEXT free channel. Furthermore since $\mathbf{H}$ is diagonally dominant no substantial power is added since the precoding matrix is almost an identity matrix.

The achievable rate on the i'th channel under no coordination is now given by [7]

$$R_i = \int_f \log_2 \left(1 + \frac{P_{s_i}(f) \left| h_{ii}(f) \right|^2}{\boldsymbol{\Gamma} \left(\sum_{l \neq i} P_{s_l}(f) \left| h_{il}(f) \right|^2 + P_{N_i}(f) \right)} \right) df \tag{5}$$

where $P_{s_i} = E\{|s_i(f)|^2\}$ is the transmit Power Spectral Density (PSD) of user i on frequency f, $P_{N_i}(f) = E\{|v_i(f)|^2\}$ is the PSD of noise. $\boldsymbol{\Gamma}$ is the gap to achievable rate which for coded QAM with BER 10^{-7} is

$9.8 - c_g + m_g$ where c_g is the coding gain and m_g is the margin. For estimating achievable rate the gap should be 0 dB ($\mathbf{\Gamma} = 1$). Assuming linear ZF precoding the crosstalk is canceled and the achievable rate is given by

$$R_i = \int_f \log_2 \left(1 + \frac{P_{s_i}(f) \left| h_{ii}(f) \right|^2}{\mathbf{\Gamma} P_{N_i}(f)} \right) df \tag{6}$$

As we will see in simulations in typical cases this leads to substantial increase in rate.

III. FEXT CANCELLATION USING PRECODING

VDSL systems typically operate over short loops and the signal to noise ratio is typically very high supporting very high achievable rate using up to 15 bits constellations over some tones. In this section we will use the assumption of good signal to noise ratio to provide a computationally simple cross-talk precoding for VDSL. As explained in the introduction joint processing can only be implemented at the ONU. Therefore FEXT cancellation can only be implemented by precompensation. The first proposal for precompensation (also named precoding or vectoring) was by Ginis and Cioffi using a Tomlinson Harashima (TH) type non-linear precoder. This analogy with the TH precoder is the source of the name precoding. Following this work it has been noted [3] that upstream VDSL channel matrices are column-wise diagonally dominant, while downstream matrices are row-wise diagonally dominant. For these kind of matrices the loss in linear FEXT precoding is small and therefore a ZF precoder achieves near optimal performance. It should be noted that VDSL systems include 4096 tones with more tones anticipated in next generation VDSL2. Therefore the complexity of inverting a 25x25 (a typical American cable) or larger matrices for each tone becomes a computationally intensive task.

In this section we propose to replace the ZF precoder by a simplified linear precoder that requires substantially less computations. The basic idea is to use the row-wise diagonal dominance of the channel matrices to approximate the inverse of the channel matrix. To that end we introduce the definitions for row-wise diagonally dominant matrices.

Definition 3.1: A matrix $H = (h_{ij})$ is strongly row-wise diagonally dominant if

$$|h_{ii}| > \sum_{j \neq i} |h_{ij}| \tag{7}$$

A matrix $H = (h_{ij})$ is weakly row-wise diagonally dominant if

$$\max_{i,j,i \neq j} \frac{|h_{ij}|}{|h_{ii}|} < 1 \tag{8}$$

We also define two useful quantities

$$\alpha_i(f) = \max_j \frac{|h_{ij}|}{|h_{ii}|}, \qquad \alpha_{\max}(f) = \max_i \alpha_i(f) \tag{9}$$

Note that for downstream scenarios, the channel matrix is row-wise diagonally dominant even in mixed length scenarios. To observe this consider a short loop of length ℓ_1 and a long loop of length ℓ_2. The FEXT into the short loop is equivalent to the equal length FEXT, and therefore is diagonally dominant. However for the longer loop the FEXT contribution can be split into a cascade of two systems: The first is a FEXT contribution into the ℓ_1 initial segment of the loop. This FEXT is further attenuated by a loop of length $\ell_2 - \ell_1$. Hence its mean response is [8]:

$$K_{FEXT} f^2 \ell_1 IL(\ell_1, f) IL(\ell_2 - \ell_1, f), \tag{10}$$

where $IL(\ell, f)$ is the typical loop insertion loss at length ℓ and frequency f and K_{FEXT} is a constant that depends on the type of cable. Since

$$IL(\ell_1, f) IL(\ell_2 - \ell_1, f) = IL(\ell_2, f)$$

as a response of a cascade of two LTI systems we obtain that the total FEXT contribution is

$$K_{FEXT} f^2 \ell_1 IL(\ell_2, f) << IL(\ell_2, f).$$

Now let $\mathbf{H}$ be the channel matrix, we can decompose it as

$$\mathbf{H} = \mathbf{D} + \mathbf{E} = \mathbf{D}\left(\mathbf{I} + \mathbf{D}^{-1}\mathbf{E}\right) \tag{11}$$

where $\mathbf{D}$ is defined by (3) and $\mathbf{E}$ is the matrix containing off diagonal elements of $\mathbf{H}$:

$$\mathbf{E} = \begin{bmatrix} 0 & h_{12} & \cdots & h_{1p} \\ h_{21} & \ddots & \ddots & \vdots \\ \vdots & \ddots & 0 & h_{(p-1)p} \\ h_{p1} & \cdots & h_{p(p-1)} & 0 \end{bmatrix}, \tag{12}$$

The ZF precoder $\mathbf{H}^{-1}\mathbf{D}$ is given by [2]

$$\mathbf{H}^{-1}\mathbf{D} = \left(\mathbf{I} + \mathbf{D}^{-1}\mathbf{E}\right)^{-1} \mathbf{D}^{-1}\mathbf{D} = \left(\mathbf{I} + \mathbf{D}^{-1}\mathbf{E}\right)^{-1}$$

Now suppose the matrix H is strongly row-wise diagonal dominant. Then by Gersgorin theorem the absolute value of all the eigenvalues of matrix $\mathbf{D}^{-1}\mathbf{E}$ are less than one [10]. Therefore the following power series expansion converges:

$$\tilde{\mathbf{H}} = \left(\mathbf{I} + \mathbf{D}^{-1}\mathbf{E}\right)^{-1} = \left(\mathbf{I} - \mathbf{D}^{-1}\mathbf{E} + \cdots + (-1)^n \left(\mathbf{D}^{-1}\mathbf{E}\right)^n + \dots\right)$$

Note however that even if the power series does not converge, we can still use the low order approximations. In the subsequent sections we will analyze the quality of the approximate precoders under the assumption of weak diagonal dominance. Note that by our assumption $\mathbf{D}^{-1}\mathbf{E}$ has small elements compared to 1. Hence the linear precoding does not cause substantial changes in the transmitted power [2]. Therefore the k^{th} order approximation is given by:

$$\tilde{\mathbf{H}}_k = \left(\mathbf{I} - \mathbf{D}^{-1}\mathbf{E} + \cdots + (-1)^k \left(\mathbf{D}^{-1}\mathbf{E}\right)^k\right) \approx \left(\mathbf{I} + \mathbf{D}^{-1}\mathbf{E}\right)^{-1} \tag{13}$$

Based on this approximation, the precoded transmitted vector $\tilde{\mathbf{H}}_k \mathbf{s}$ is transmitted through the channel and the received vector $\mathbf{x}$ is given by

$$\mathbf{x} = \mathbf{H}\tilde{\mathbf{H}}_k\mathbf{s} + \boldsymbol{\nu} = \mathbf{D}\left(\mathbf{I} + \mathbf{D}^{-1}\mathbf{E}\right)\tilde{\mathbf{H}}_k\mathbf{s} + \boldsymbol{\nu} \tag{14}$$

Substituting (13) into (14) we obtain, using the telescopic cancellation of terms, that the received k'th order precoded symbol is given by

$$\mathbf{x} = \left(\mathbf{D} + (-1)^k \mathbf{E}\left(\mathbf{D}^{-1}\mathbf{E}\right)^k\right)\mathbf{s} + \boldsymbol{\nu} \tag{15}$$

Now the i'th receiver equalizes its signal using $\mathbf{D}^{-1}$ resulting in

$$z_i = D_{ii}^{-1}\left[\left(\mathbf{D} + (-1)^k \mathbf{E}\left(\mathbf{D}^{-1}\mathbf{E}\right)^k\right)\mathbf{s}\right]_i + D_{ii}^{-1}\nu_i. \tag{16}$$

This linear equalization is sufficient due to the DMT operation and assuming that the channel is shorter than the length of the cyclic prefix. The residual crosstalk is now a k'th order expression which we further analyze in the next section for k=1 and 2. These values correspond to first and second order approximations. For first order precoding, we have

$$\mathbf{x} = \left(\mathbf{D} - \mathbf{E}\mathbf{D}^{-1}\mathbf{E}\right)\mathbf{s} + \boldsymbol{\nu} \tag{17}$$

and second order precoding

$$\mathbf{x} = \left(\mathbf{D} + \mathbf{E}\left(\mathbf{D}^{-1}\mathbf{E}\right)^2\right)\mathbf{s} + \boldsymbol{\nu} \tag{18}$$

Note that the precoding matrix is a polynomial in $\mathbf{D}^{-1}\mathbf{E}$. This implies that the multiplication of the precoder by the data vector can be implemented using Horner's formula. This will be explained in section VI.

In the next sections we will analyze these precoding schemes analytically and in simulations based on measured FEXT and channel transfer functions.

IV. Performance analysis of the proposed precoding method

We turn now to analyze the proposed first and second order approximations to the ZF precoder given by (17), (18). In this section we provide simplified bounds on the achievable rate of the simplified precoders, and evaluate the loss compared to the optimal ZF precoders. The analysis of the first order precoder will be given in the main text, while only the final bounds for the second order precoder are provided due to space limitations. As a first step we would like to evaluate the received PSD of the desired signal at the $i'th$ receiver and the corresponding noise and residual FEXT at the receiver. As usual we perform the analysis assuming optimal Gaussian signaling and then incorporate a Shannon gap approximation to obtain the performance under non-ideal channel coding [6]. Hence we can assume Gaussian signaling, and the residual interference due to the non-ideal precoding is also Gaussian. Since the Gaussian interference is the worst case the bound remains valid also when other types of modulation are used. Let

$$\boldsymbol{\Delta}_{il}(f) = \sum_{k\neq i,l} \frac{h_{ik}(f)h_{kl}(f)}{h_{kk}(f)} \quad i \neq l, \quad \boldsymbol{\Delta}_{ii}(f) = \sum_{k\neq i} \frac{h_{ik}(f)h_{ki}(f)}{h_{kk}(f)} \tag{19}$$

be the elements of $\mathbf{E}\mathbf{D}^{-1}\mathbf{E}$. The received i'th signal PSD at the i'th receiver is therefore given by

$$P_{sig}(f,i) = P_{s_i}(f)|h_{ii}(f) - \boldsymbol{\Delta}_{ii}(f)|^2 \tag{20}$$

Similarly the received residual FEXT plus noise PSD at the i'th receiver is given by:

$$P_{noise}(f,i) = P_{N_i}(f) + \sum_{l\neq i} P_{s_l}(f)\left|\boldsymbol{\Delta}_{il}(f)\right|^2 \tag{21}$$

We can now state the performance bound on the accuracy of the first order precoding

Theorem 4.1: Let $\mathbf{H}(f) = (h_{ij}(f))$ be a weakly row-wise diagonally dominant channel transfer matrix. Then the total achievable rate for channel i when using first order precoding and signaling with a Shannon gap of Γ is given by

$$R_{FO_i} = \sum_k R_{FO_i}(f_k) = \Delta f \sum_k \log_2\left(1 + \frac{P_{sig}(f_k,i)}{\Gamma P_{noise}(f_k,i)}\right). \tag{22}$$

Furthermore the achievable rate $R_{FO_i}(f_k)$ at tone k and spatial channel i is lower bounded by:

$$R_{FO_i}(f_k) \geq \Delta f \log_2\left(1 + SN_{FO_i}(f_k)\right)$$

where

$$SN_{FO_i}(f_k) = \frac{h^2_{i,\text{eff}}(f_k)P_{s_i}(f_k)}{\Gamma\left(P_{N_i}(f_k) + g^2_{i,\text{eff}}(f_k)\sum_{l\neq i}P_{s_l}(f_k)\right)} \tag{23}$$

$$\begin{aligned} h^2_{i,\text{eff}}(f_k) &= \left(1-(p-1)\alpha^2_{max}(f_k)\right)|h_{ii}(f_k)|^2 \\ g^2_{i,\text{eff}}(f_k) &= (p-2)^2\alpha^4_{max}(f_k)|h_{ii}(f_k)|^2. \end{aligned} \tag{24}$$

Before proving the theorem we would like to emphasize that the bound in theorem depends only on α_{max} and not on $\sum_{j\neq i}\frac{|h_{ij}|}{|h_{ii}|}$. Hence we do not need the convergence of the power series in order to utilize the bound or the precoder, so weak diagonal dominance suffices. We now turn to the proof of the theorem.

Proof: Writing (17), explicitly the received signal at the i'th receiver is given by

$$x_i(f) = h_{ii}(f)s_i(f) - \sum_{l=1}^{p}\mathbf{\Delta}_{il}(f)s_l(f) + \nu_i(f) = h'_{ii}(f)s_i(f) + n'_i(f) \tag{25}$$

where $n'_i(f)$ is the total noise and residual crosstalk at the i'th channel

$$n'_i(f) = -\sum_{l\neq i}\mathbf{\Delta}_{il}(f)s_l(f) + \nu_i(f) \tag{26}$$

and $h'_{ii}(f) = h_{ii}(f) - \mathbf{\Delta}_{ii}(f)$. By definition of $\alpha_i(f)$ for all $i \neq l$

$$|h_{il}(f)| \leq \alpha_i(f)|h_{ii}(f)| \tag{27}$$

Applying Cauchy inequality to (19) we obtain

$$\begin{aligned} &|\mathbf{\Delta}_{il}(f)|^2 = |\textstyle\sum_{k\neq i,l}\frac{h_{ik}(f)h_{kl}(f)}{h_{kk}(f)}|^2 \\ &\leq \left(\textstyle\sum_{k\neq i,l}\frac{|h_{ik}(f)|^2}{|h_{ii}(f)|^2}\right)\left(\textstyle\sum_{k\neq i,l}\frac{|h_{kl}(f)|^2}{|h_{kk}(f)|^2}\right)|h_{ii}(f)|^2 \\ &\leq a_1(f,i,l)|h_{ii}(f)|^2 \end{aligned} \tag{28}$$

where $a_1(f,i,l) = (p-2)\alpha_i^2(f)\sum_{k\neq i,l}\alpha_k^2(f)$. Similarly $|\mathbf{\Delta}_{ii}(f)| \leq a_2(f,i)|h_{ii}|$ where

$$a_2(f,i) = (p-1)^{\frac{1}{2}}\alpha_i(f)\left(\sum_{k\neq i}\alpha_k^2(f)\right)^{\frac{1}{2}}$$

Therefore

$$\begin{aligned} &|h'_{ii}(f)|^2 = |h_{ii}(f) - \mathbf{\Delta}_{ii}(f)|^2 \geq (|h_{ii}(f)| - |\mathbf{\Delta}_{ii}(f)|)^2 \\ &\geq (1-a_2(f,i))^2|h_{ii}(f)|^2 \end{aligned} \tag{29}$$

Hence the signal and noise plus residual cross-talk PSD can be bounded respectively by

$$P_{sig}(f,i) \geq (1-a_2(f,i))^2|h_{ii}(f)|^2P_{s_i}(f) \tag{30}$$

$$P_{noise}(f,i) \leq P_{N_i}(f) + |h_{ii}(f)|^2\sum_{l\neq i}a_1(f,i,l)P_{s_l}(f) \tag{31}$$

The achievable rate of a specific frequency bin $f_k \leq f \leq f_k + \Delta f$ with gap Γ [6] for channel i is bounded by

$$\begin{aligned} &R_{FO_i}(f_k) = \Delta f\log_2\left(1 + \frac{P_{sig}(f_k,i)}{\Gamma P_{noise}(f_k,i)}\right) \\ &\geq \Delta f\log_2\left(1 + \frac{(1-a_2(f_k,i))^2|h_{ii}(f_k)|^2P_{s_i}(f_k)}{\Gamma\left(P_{N_i}(f_k) + |h_{ii}(f_k)|^2\sum_{l\neq i}a_1(f_k,i,l)P_{s_l}(f_k)\right)}\right) \end{aligned} \tag{32}$$

Further simplification using (9) and assuming $(p-1)\alpha_{max}(f_k)^2 < 1$ yields

$$R_{FO_i}(f_k) \geq \Delta f\log_2\left(1 + \frac{h^2_{i,\text{eff}}P_{s_i}(f_k)}{\Gamma\left(P_{N_i}(f_k) + g^2_{i,\text{eff}}\sum_{l\neq i}P_{s_l}(f_k)\right)}\right) \tag{33}$$

The bound on the achievable rate for a single channel is therefore given by

$$\sum_k R_{FO_i}(f_k) \geq \sum_k \Delta f \log_2 \left(1 + SN_{FO_i}(f_k)\right) \tag{34}$$

This ends the proof.

To gain some more insight into the bound we would like to obtain some simplified forms of the bound. Simple algebraic manipulations on (33) yield

$$\begin{aligned} R_{FO_i}(f_k) \geq \quad & \Delta f \log_2 \left(\frac{\text{SNR}(f_k,i)}{\Gamma}\right) \\ & -\Delta f \log_2 \left(1 + (p-2)^2 \alpha_{max}^2(f_k) \text{INR}(f_k,i)\right) \\ & +\Delta f \log_2 \left(1 - (p-1)\alpha_{max}^2(f_k)\right)^2 \end{aligned} \tag{35}$$

where

$$\begin{aligned} \text{SNR}(f_k,i) &= \frac{P_{s_i}(f_k)|h_{ii}(f_k)|^2}{P_{N_i}(f_k)}, \\ \text{INR}(f_k,i) &= \frac{\alpha_{max}^2(f_k)|h_{ii}(f_k)|^2 \sum_{l\neq i} P_{s_l}(f_k)}{P_{N_i}(f_k)} \end{aligned} \tag{36}$$

are respectively the signal to noise ratio and a bound on the total residual FEXT to noise ratio. Note that for the special case of equal length and symmetric channel this is indeed the interference to noise ratio at the i'th receiver.

In DSL typically the FEXT noise is much stronger than the AWGN (up to 20 dB) so even when the SINR is high there is substantial gain in performing FEXT cancellation. When no cancellation is performed and the SINR is high we obtain

$$R_{NC}(f_k,i) \approx \Delta f \log_2 \left(\frac{\text{SNR}(f_k,i)}{\Gamma}\right) - \Delta f \log_2 \left(1 + \text{INR}(f_k,i)\right) \tag{37}$$

Therefore the additional spectral efficiency due to first order crosstalk cancellation is approximated by

$$\begin{aligned} \Delta R_{FO}(f_k) \quad & \approx R_{FO_i}(f_k) - R_{NC}(f_k) \\ & = -\Delta f \log_2 \left(\frac{1+(p-2)^2\alpha_{max}^2(f_k)\text{INR}(f_k,i)}{1+\text{INR}(f_k,i)}\right) \\ & \quad +\Delta f \log_2 \left(1 - (p-1)\alpha_{max}^2(f_k)\right)^2 \end{aligned} \tag{38}$$

To gain some insight into the result consider the case of high INR, i.e., the FEXT limited scenario. In this case we can approximate the achievable rate gain by

$$\begin{aligned} \Delta R_{FO}(f_k,i) \quad & \geq -\Delta f \log_2 \left((p-2)^2 \alpha_{max}^2(f_k)\right) \\ & \quad +\Delta f \log_2 \left(1 - (p-1)\alpha_{max}^2(f_k)\right)^2 . \end{aligned} \tag{39}$$

The gain is positive as long as $\alpha_{max}(f_k) < \frac{1}{\sqrt{p-1}}$, assumption that holds as long as the signal to FEXT ratio is above 7dB [1].

The above analysis demonstrates that performance of first order precoding depends on $\alpha_{max}(f)$. If this value is small, the proposed precoding approaches the optimal processing, otherwise, the loss is apparent. However using (32) might yield tighter bound, although more complicated. We will show in simulations based on both theoretical models and measured data that the assumptions hold under wide range of scenarios. Furthermore, since the precoder can estimate in advance all the crosstalk coupling functions, it can use optimal matrix inversion only at tones where this matrix inversion is necessary.

We now describe the extension of the results to the case of second order precoding.

[1]Practically (see [11]) when the assumption is violated the SNR is below 10 dB and therefore the achievable rate on the specific frequency bin is basically 0 taking into account implementation loss and 6 dB noise margin.

Let $\mathbf{O} = \left(\mathbf{D}^{-1}\mathbf{E}\right)^2 = (o_{ij})$ $\boldsymbol{\Omega} = \mathbf{E}\left(\mathbf{D}^{-1}\mathbf{E}\right)^2 = (\Omega_{ij})$, then we have

$$o_{ii}(f) = \sum_{l \neq i} \frac{h_{il}(f)h_{li}(f)}{h_{ii}(f)h_{ll}(f)}, \quad o_{ij}(f) = \sum_{l \neq i,j} \frac{h_{il}(f)h_{lj}(f)}{h_{ii}(f)h_{ll}(f)} \quad i \neq j \tag{40}$$

and

$$\Omega_{ij}(f) = \sum_{l \neq i} h_{il}(f) o_{lj}(f) \tag{41}$$

Similarly to the first order case the received signal and noise plus residual cross-talk power for channel i are defined respectively as

$$P_{sig}(f,i) = P_{s_i}(f)|h_{ii}(f) + \Omega_{ii}(f)|^2 \tag{42}$$

and

$$P_{noise}(f,i) = P_{N_i}(f) + \sum_{j \neq i} P_{s_j}(f)\left|\Omega_{ij}(f)\right|^2. \tag{43}$$

A lower bound on the achievable rate is now given by the following theorem.

Theorem 4.2: Let $\mathbf{H}$ be a weakly row-wise diagonally dominant channel matrix. The total achievable rate for channel i when using second order precoding matrix is given by

$$R_{SO_i} = \Delta f \sum_k \log_2\left(1 + \frac{P_{sig}(f_k,i)}{\Gamma P_{noise}(f_k,i)}\right).$$

Furthermore, the following formula provides a lower bound for the achievable rate:

$$R_{SO_i} \geq \sum_{k=1}^{K} \log_2\left(1 + \frac{P_{SO}^{rx}(f_k)}{\Gamma N_{SO}^{rx}(f_k)}\right) \tag{44}$$

where

$$P_{SO}^{rx}(f_k) = |h_{ii}(f_k)|^2 \left(1 - (p-1)(p-2)\alpha_{max}^3(f_k)\right)^2 P_{s_i}(f_k)$$

and

$$N_{SO}^{rx}(f_k) = P_{N_i}(f_k) + \xi \alpha_{max}^6(f_k)|h_{ii}(f_k)|^2 \textstyle\sum_{j \neq i} P_{s_j}(f_k).$$

where $\xi = (p-1)\left((p-1)^2 + (p-2)^3\right)$

The proof of this theorem is not included due to space limitations, but the techniques used are similar to the ones used in theorem 4.1

V. Performance Comparisons

In the first experiment we evaluate the performance of the proposed method through France Telecom experimental data of 8 VDSL lines and a system composed of 2 lines at each distance (75m, 150m, 300m, and 590m). Each modem has a coding gain of 3.8 dB, noise margin of 6 dB which results in a gap Γ with $10\log_{10}\Gamma = 12dB$ for BER of 10^{-7}. We have performed 1000 independent runs, and in each run, the insertion loss and FEXT are taken randomly from the 28 pairs of the given length. The crosstalk coupling functions have been computed by cascading the equal length FEXT transfer functions for the shorter pairs and the insertion loss for the remaining length of the cable. Figure 1 depicts estimated CDFs of the achievable rates of the four methods when the extended VDSL frequency band (called extended VDSL 998) used are 0.138-3.75MHz, 5.2-8.5MHz, and 12-20MHz. We can clearly see that the second order precoding has performance almost identical to the performance of the linear

ZF precoder over all loops. The first order precoder suffers from a loss over the shorter loops of 75m and 150m, but has almost no loss at distances above 300m. The first order precoder provides rates of 100Mbps over 600m and 200 Mbps over loops of 300m which doubles the performance of the non-precoded system. We can clearly

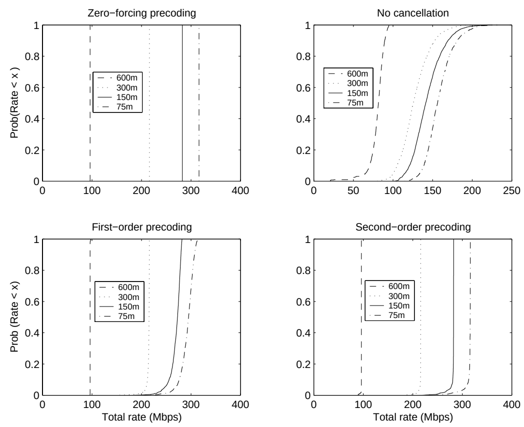


Fig. 1. Achievable rate distribution of 8 pair systems. Extended VDSL 998 band plan with DS_3 =[12, 20]MHz.

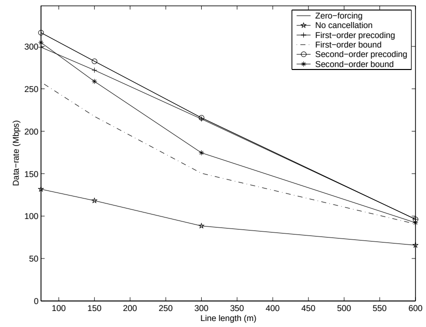
see the good performance of the second order precoder at outage level of 1%. The first order precoder has good average behavior but suffers about 10% rate loss for the 150m loops.

In the next experiment, we used the measured France Telecom data with equal length loops. We compare performance over 75m-600m loops with masks identical to the first scenario, when using four pairs of the same length. However the lines were chosen according to the worst case interference scenario were same quad interference was present for all loops. Figure 2(a) presents the results using extended VDSL frequency allocation. All other conditions are the same as in the first experiment. From this figure we see that second-order precompensation leads to performance identical to ZF, while some loss for the first-order precompensation is apparent for very short loop lengths. However this is the price paid for reduced computational complexity. It is interesting to note that rates of 250 Mbps per channel are achievable for loops of 200m [14]. Also note that due to the worst case noise conditions the simplified bounds are less tight than in the simulated data which represents average disturbance.

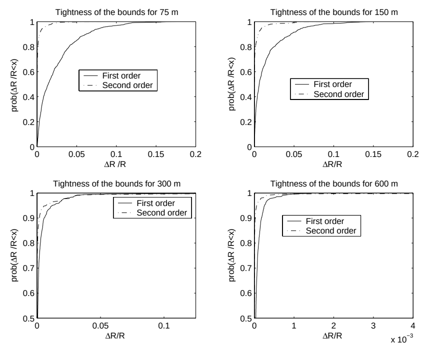
Finally we have tested the tightness of the bound in the case of mixed length. It is anticipated that in this case the bound will be less tight due to the overestimation of the residual FEXT power. We have performed 1000 Monte-Carlo trials. In each trial we have randomly picked 4 lines at 75, 150, 300, and 600m. We have computed the capacity based on first order and second order precoders as well as the corresponding bound. The usefulness of the bound is measured by the relative error. Define $\Delta R = R_{\text{precoder}} - R_{\text{bound}}$. The tightness parameter is now given by

$$\varepsilon_r = \frac{\Delta R}{R_{\text{precoder}}} \tag{45}$$

Figure 2(b) depicts the tightness of the bound for each of the lines. The solid lines represent the tightness of the first order bound, while the dashed lines represent the tightness of the second order bound. We can see that for 90% of the cases the first order bound had a relative error of less than 7%, while the second order bound had less than 1% relative error. Moreover, for the 300m and 600m lines the first order bound is also very tight. This implies that the bounds indeed provided a simple mean to estimate the achievable rate using the relevant precoding technique.



(a)



(b)

Fig. 2. (a) 4 lines, measured, extended (VDSL2) bandplan. (b) Tightness of the bounds for 4 lines at different distances (75, 150, 300 and 590m). Solid lines: First order bound. Dashed lines: Second order bound.

VI. Complexity of the proposed method

In this section we analyze the complexity of the proposed method. The complexity analysis is done in two steps: First we analyze the initialization of the precoder and then the steady state complexity. The starting point for the analysis is (13) assuming $k = 2$. Using Horner's formula for the second order precoder, we can rewrite (13) as

$$\tilde{\mathbf{H}}_2 = \mathbf{I} - \mathbf{D}^{-1}\mathbf{E}\left(\mathbf{I} - \mathbf{D}^{-1}\mathbf{E}\right) \tag{46}$$

Hence the precoder needs to compute at each step

$$\mathbf{x} = \tilde{\mathbf{H}}_2\mathbf{s} = \mathbf{s} - \mathbf{D}^{-1}\mathbf{E}\left(\mathbf{s} - \mathbf{D}^{-1}\mathbf{E}\mathbf{s}\right) \tag{47}$$

Therefore we will not compute $\mathbf{D}^{-1}\mathbf{E}$ but at initialization we will only store the off-diagonal channels and invert the diagonal elements. Assuming p operating lines and K frequency bands the initialization now reduces to pK scalar inversions. This is instead of K matrix inversions, requiring p^3K operations, and p^2K memory words. The complexity saving is a factor of p^2 at initialization. With $p = 25$ this is a factor of 625. Since in many DSL modems initialization is performed by a strong DSP this also reduces the requirements for this DSP as well as the extra program and data memory required to invert matrices. Furthermore, $\mathbf{D}^{-1}$ can be obtained as the coefficient of the frequency domain equalizer, when used in a ZF mode, so each receiver estimates and updates it as part of its normal mode of operation. Hence each receiver needs only multiply the FEQ coefficient by its own local version of the relevant column of the channel matrix, and only the off diagonal elements are sent over the channel to the transmitter. The estimated silicon savings in 90nm technology are 10mm[2] (trading the additional memory

[2]The authors would like to thank A. Gal from Metalink Broadband for estimating the total Silicon area savings

requirements with the required multipliers). These are very substantial cost savings, while the increase in arithmetic complexity is moderate (taking into account that only tones with low SNR should use the second order precoding, while most tones will use the first order precoder. The bounds provided in this paper also serve the purpose of choosing the correct precoding scheme for each tone.

VII. Conclusions

In this paper we have demonstrated the effectiveness of linear ZF precoding using approximate inverses. These approximate inverses are significantly simpler than ideal ZF precoders. Furthermore we can see that for 200m loops full binder coordination will lead to rates close to 250 Mbps over a single loop or 1 Gbps over 4 loops. While the way to practical coordination of a full binder is quite long, this paper alleviates much of the complexity involved in the precoding step. We have shown great reduction in all aspects of initialization complexity, and how we are able to trade large memory requirements with moderate increase in computational complexity.

Acknowledgment

We would like to thank Rabah Tarafi, Frederic Gauthier and Hubert Mariotte from FT R&D labs for providing crosstalk measurements that have been used in this paper, and to N. Sidiropoulos for comments on earlier drafts of this paper. We would also like to thank the anonymous reviewers for their helpful suggestions significantly improving the presentation of the paper.

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