Frequency Assignment

Radio interface planning in mobile telecommunication

Martin Grötschel

Summer School

Progress in Mathematics for Communication Systems

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plus several MSc students

About this presentation

- This talk is based on work of many years of the ZIB telecom research group. Input, in particular, from
  - Andreas Eisenblätter
  - Hans-Florian Geerdes
  - Arie Koster
  - Thorsten Koch

is gratefully acknowledged.

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ZIB Partners from Industry

- Bell Communications Research (now Telcordia)
- Telenor (Norwegian Telecom)
- E-Plus
- DFN-Verein
- Bosch Telekom (bought by Marconi)
- Siemens
- Austria Telekom
- T-Systems Nova (Deutsche Telekom)
- KPN
- Telecel-Vodafone

- Atesio (ZIB spin-off company)

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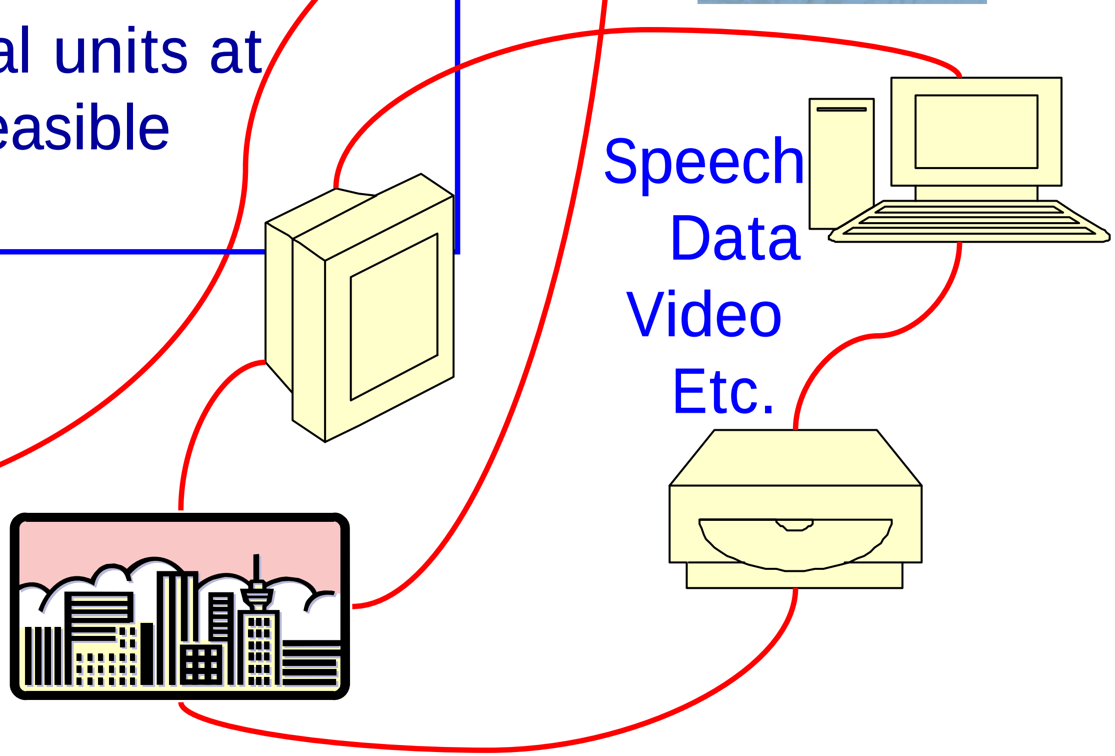
What is the Telecom Problem?

Design excellent technical devices and a robust network that survives all kinds of failures and organize the traffic such that high quality telecommunication between very many individual units at many locations is feasible at low cost!



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What is the Telecom Problem?

Design excellent technical devices and a robust network that survives all kinds of failures and organize the traffic such that high quality telecommunication between very many individual units at many locations is feasible at low cost!

This problem is too general to be solved in one step.

Approach in Practice:

- Decompose whenever possible.
- Look at a hierarchy of problems.
- Address the individual problems one by one.
- Recompose to find a good global solution.

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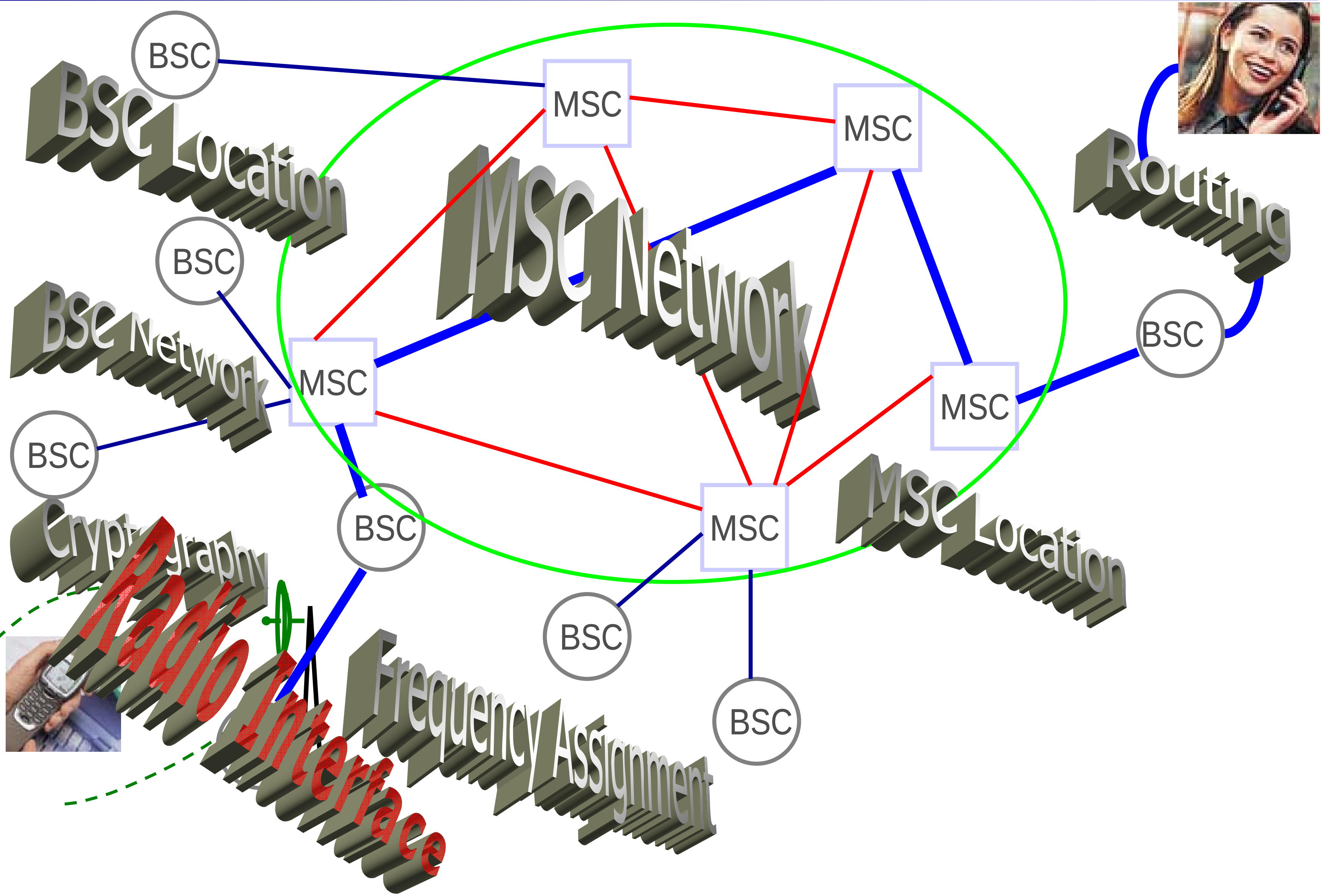
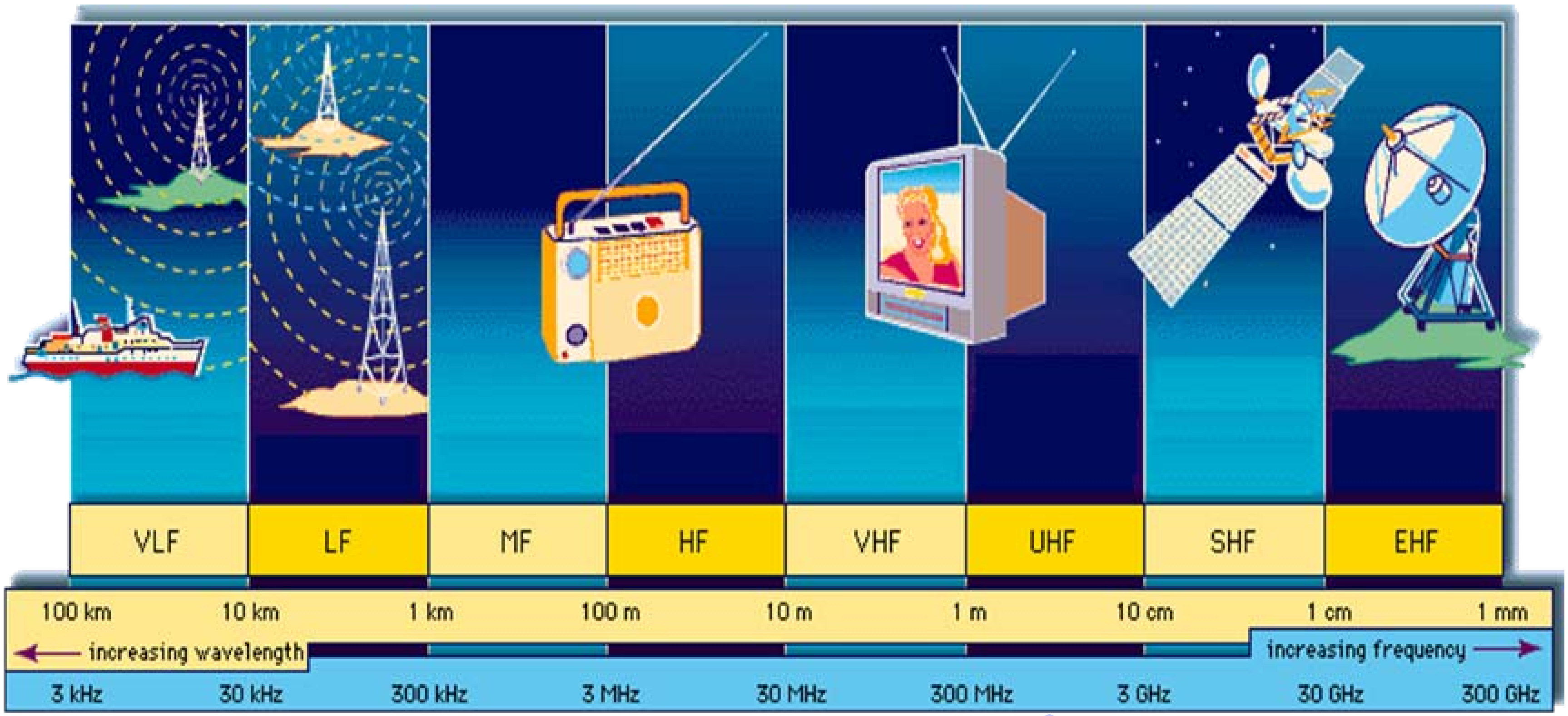
Connecting Mobiles: What´s up?



Wireless Communication



Mobile Telecommunication

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Generations of Mobile Telecommunications Systems

1G	2G	3G	4G
1980s	1990s	2000s	2010 ??

- Analogue
- Voice Only

- Digital
- Voice & Data
- **GSM** mass market
- PCS
- cdmaOne/IS95

- **UMTS**, WiFi/WLAN, cdma2000
- Data Rates ≥ 384 kbit/s
- Various Services

- more **services**
- more bandwidth
- fresh spectrum
- new technology
- **W-CDMA** radio transmissions

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Radio Interface: OR & Optimization Challenges

- Location of sites/base stations
  - was investgated in the OR literature („dead subject“)
  - has become „hot“ again
    - UMTS: massive investments around the world
    - GSM: still significant roll-outs
    - special issue: mergers
- antenna configurations at base stations
  - GSM: coverage based planning
  - UMTS: coverage & capacity considerations
- radio resource allocation
  - GSM: frequency assignment
  - UMTS: ? (open: real time/online resource management)

Contents

Wireless Communication

GSM Frequencies: 450, 900, 1800, 1900 MHz

More than 500 million users in over 150 countries

FAP
F
i
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m

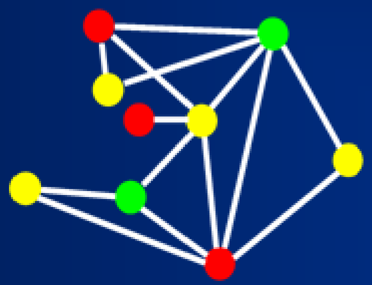
Antennas

Initial Idea

- Use graph colouring to assign channels!

Coloring Graphs

Given a graph G = (V,E), color the nodes of the graph such that no two adjacent nodes have the same color.



The smallest number of colors with this property is called chromatic or coloring number and is denoted by $\chi(G)$.

Coloring Graphs

A typical theoretical question: Given a
class *C* of graphs
(e.g., planar or perfect graphs, graphs without certain minors), what can one prove about the chromatic number of all graphs in *C* ?

A typical practical question: Given a
particular graph G
(e.g., arising in some application), how can one determine (or approximate) the chromatic number of G?

Coloring Graphs

- Coloring graphs algorithmically

  - NP-hard in theory
  - very hard in practice
  - almost impossible to find optimal colorings (symmetry issue)
  - playground for heuristics (e.g., DIMACS challenge)

Coloring in Telecommunication

- Frequency or Channel Assignment for radio-, tv-transmission, etc.
- Our Example: GSM mobile phone systems

- Andreas Eseinblätter, Martin Grötschel and Arie M. C. A. Koster, *Frequenzplanung im Mobilfunk*, DMV-Mitteilungen 1(2002)18-25
- Andreas Eisenblätter, Hans-Florian Geerdes, Thorsten Koch, Ulrich Türke: *MOMENTUM Data Scenarios for Radio Network Planning and Simulation* , ZIB-Report 04-07
- Andreas Eisenblätter, Armin Fügenschuh, Hans-Florian Geerdes, Daniel Junglas, Thorsten Koch, Alexander Martin: *Optimization Methods for UMTS Radio Network Planning*, ZIB-Report 03-41

Properties of wireless communication

detects
oscillations

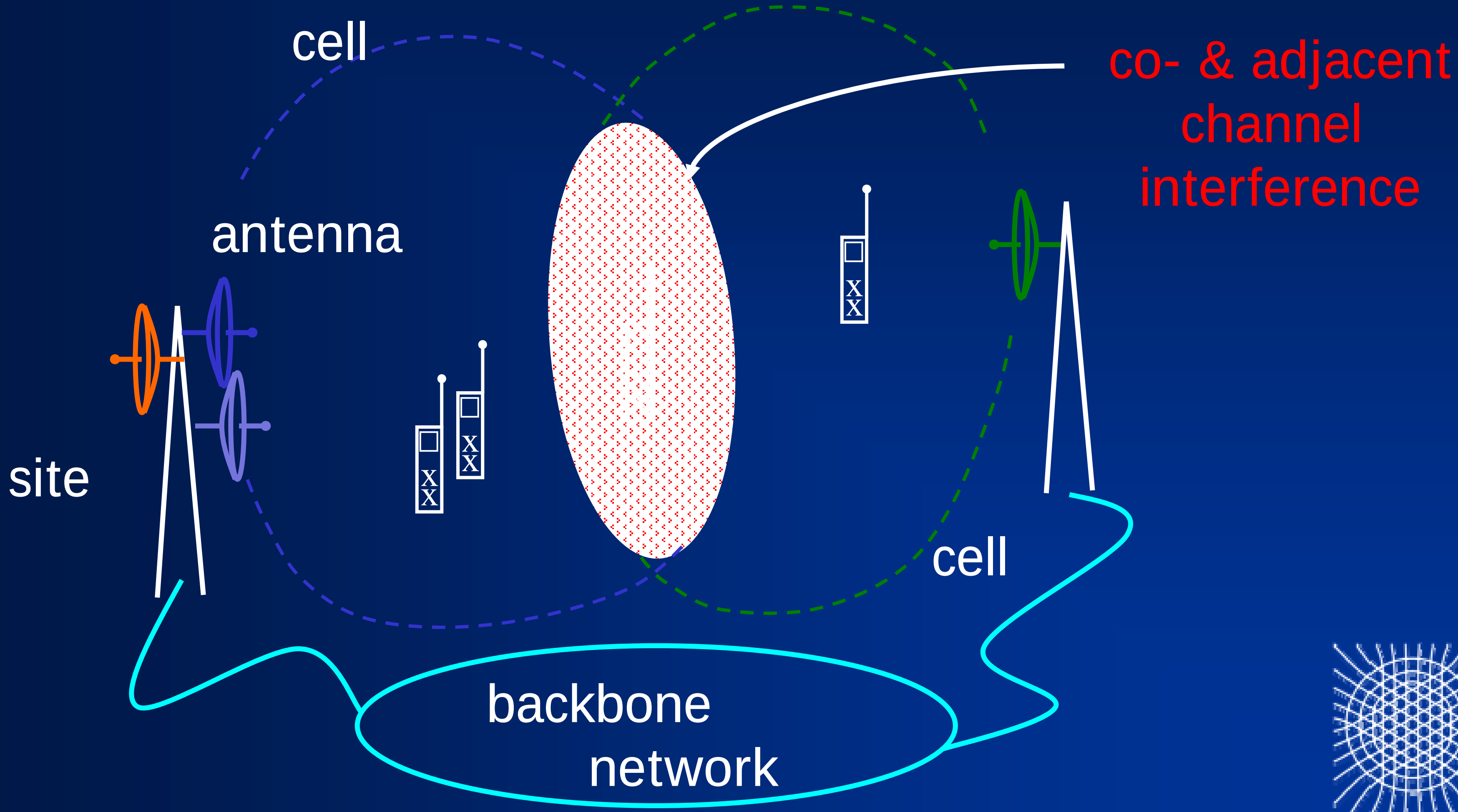
emits electromagnetic
oscillations at a frequency

Quality of the received signal:
Signal-to-noise ratio

Poor signal-to-noise ratio:
interference of the signal

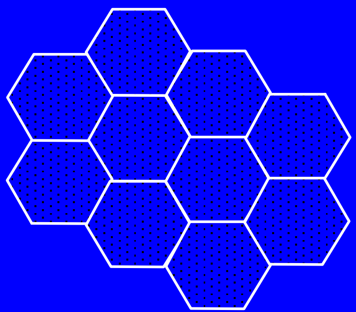
Objective: Frequency plan without interference or,
second best, with minimum interference

Antennas & Interference



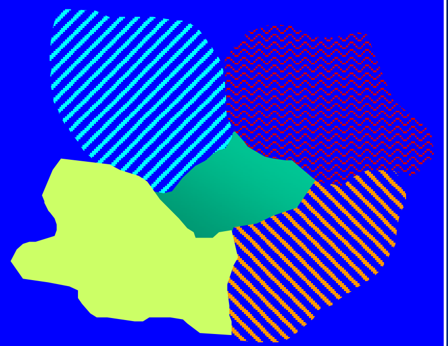
Cell Models

Hexagon Cell Model



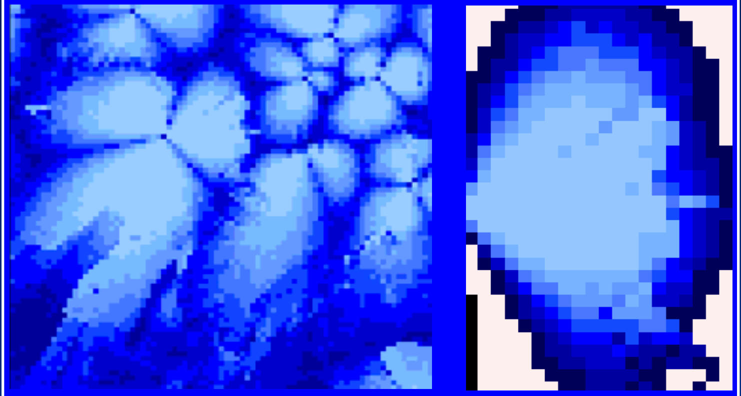
- **sites on regular grid**
- **isotropic propagation conditions**
- **no cell-overlapping**

Best Server Model



- **realistic propagation conditions**
- **arbitrary cell shapes**
- **no cell-overlapping**

Cell Assignment Probability Model



- **realistic propagation conditions**
- **arbitrary cell shapes**
- **cell-overlapping**

Source: **E-Plus Mobilfunk, Germany**

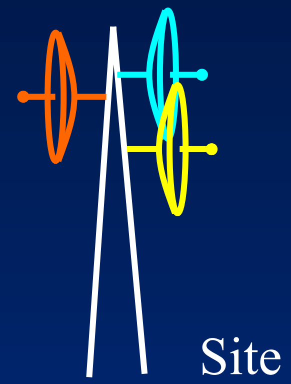
Interference

Level of interference depends on

- distance between transmitters,
- geographical position,
- power of the signals,
- direction in which signals are transmitted,
- weather conditions
- assigned frequencies
  - co-channel interference

Separation

Frequencies assigned to the same location (site) have to be separated



Blocked channels

Restricted spectrum at some locations:

- government regulations,
- agreements with operators in neighboring regions,
- requirements of military forces,
- etc.

Frequency Planning Problem

Find an assignment of frequencies/channels to transmitters that satisfies

- all separation constraints
- all blocked channels requirements

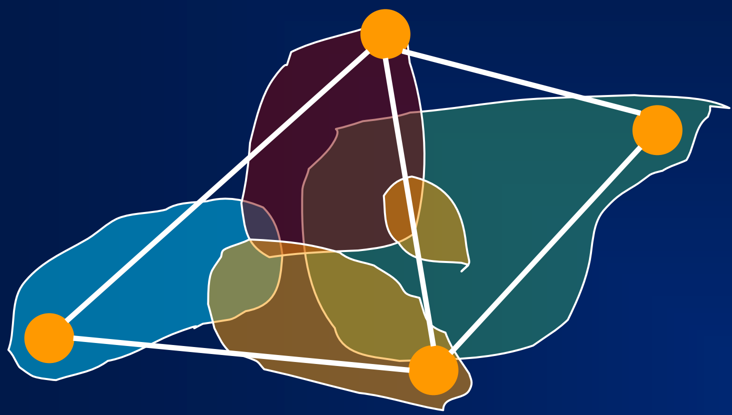
and either

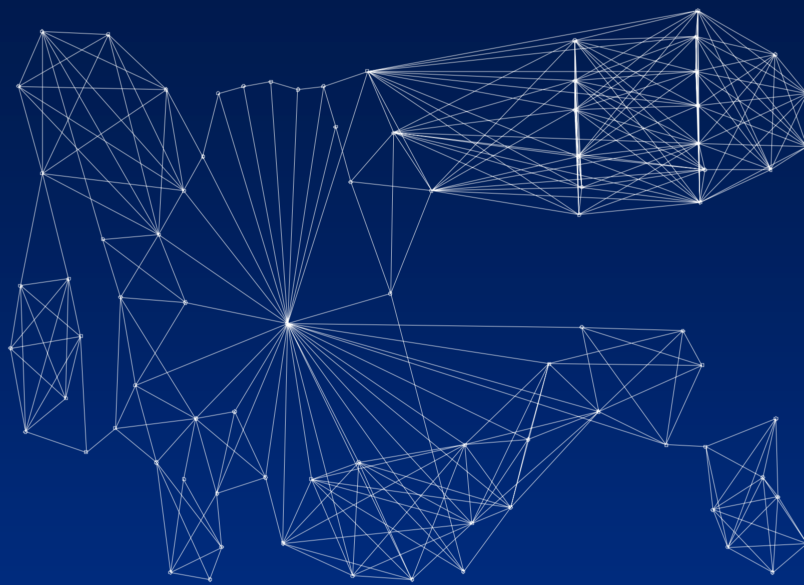
- avoids interference at all

or

- minimizes the (total/maximum) interference level

Modeling: the interference graph





- Vertices represent transmitters (TRXs)
- Edges represent separation constraints and co/adjacent-channel interference
  - Separation distance: $d(vw)$
  - Co-channel interference level: $c^{co}(vw)$
  - Adjacent-channel interference level: $c^{ad}(vw)$

Remark about UMTS

- There is no way to model interference as some number associated with an edge in some graph.
- Modelling is much more complicated,
a glimpse will be given later.

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Graph Coloring

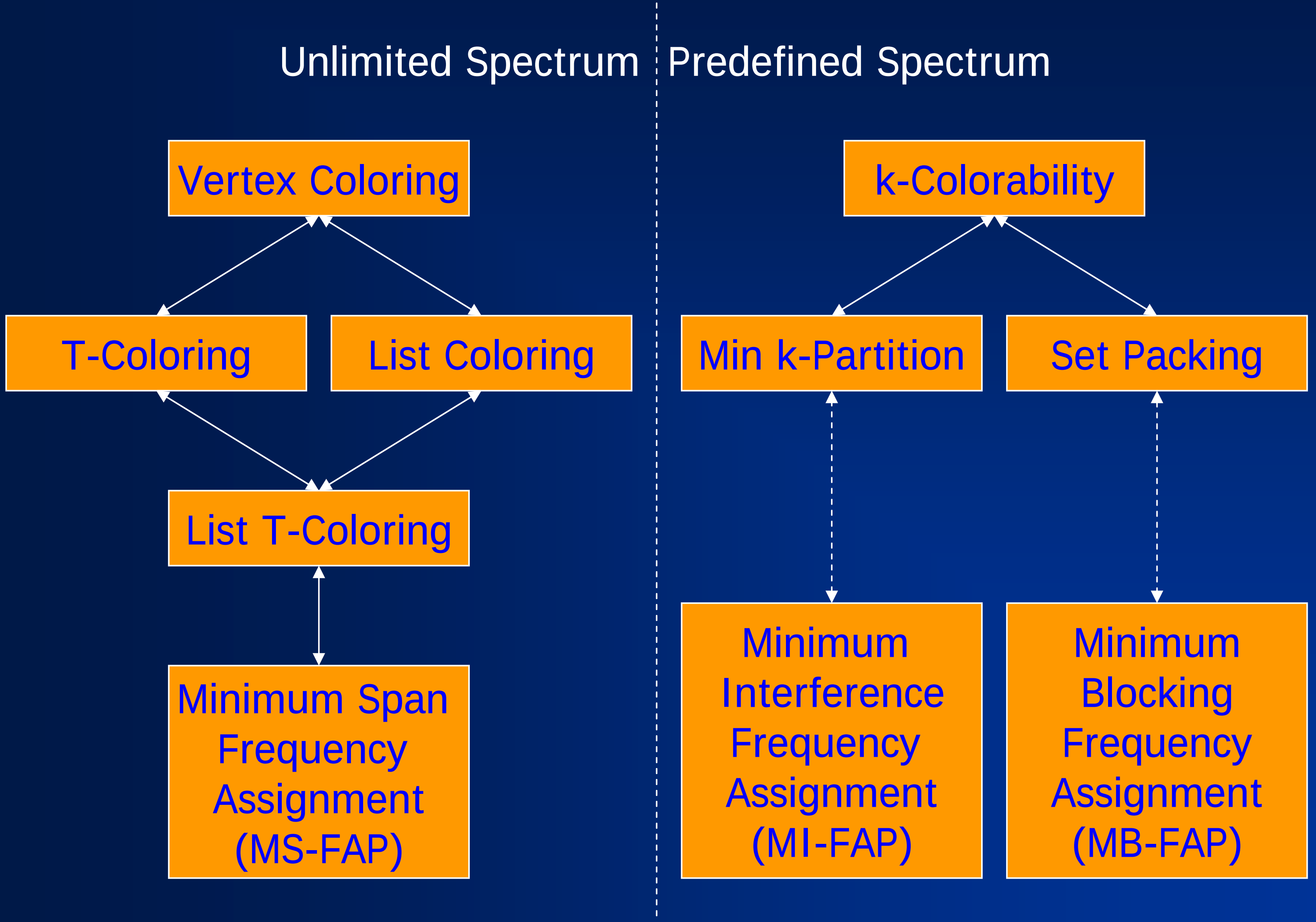
Simplifications:

- drop adjacent-channel interference
- drop local blockings
- reduce all separation requirements to 1
- change large co-channel interference into separation distance 1 (inacceptable interference)

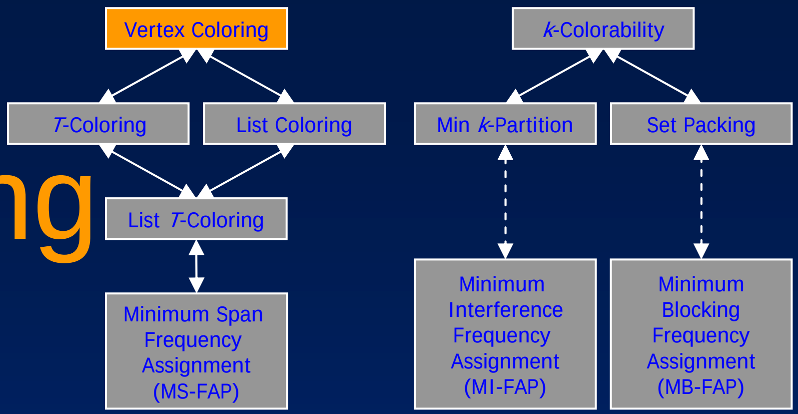
Result:

- FAP reduces to coloring the vertices of a graph

Graph Coloring & Frequency Planning

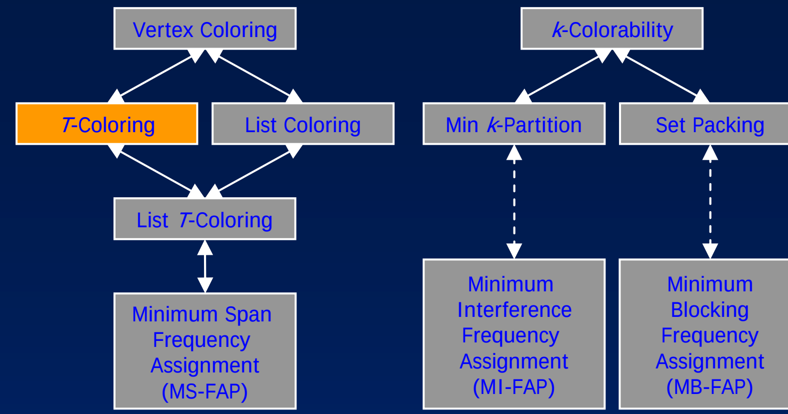


FAP & Vertex Coloring



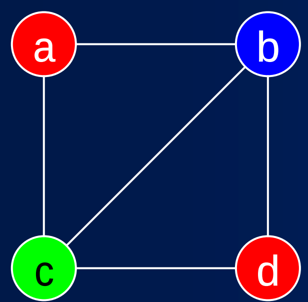
- Only co-channel interference
- Separation distance 1
- Minimization of
  - Number of frequencies used (chromatic number)
  - Span of frequencies used
- Objectives are equivalent: span = #colors-1
- FAP is **NP**-hard

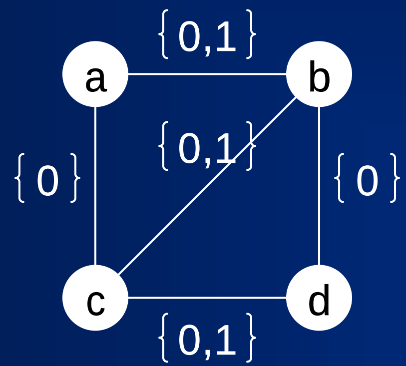
FAP & T-Coloring

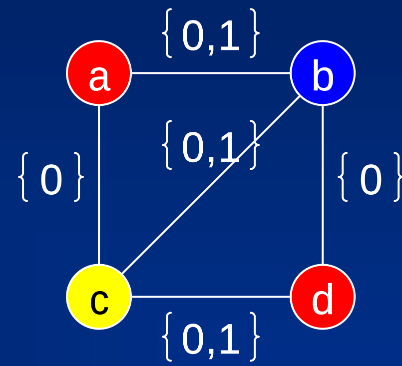


Sets of forbidden distances T_{vw}

$$|f_v - f_w| \notin T_{vw} \qquad T_{vw} = \{0,\ldots,d(vw)\text{-}1\}$$

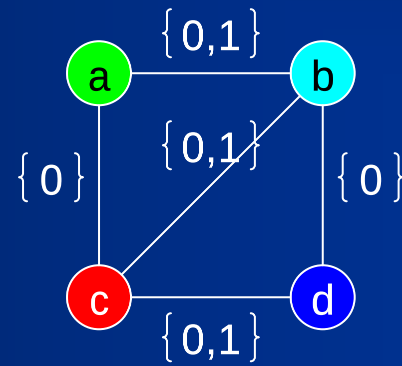






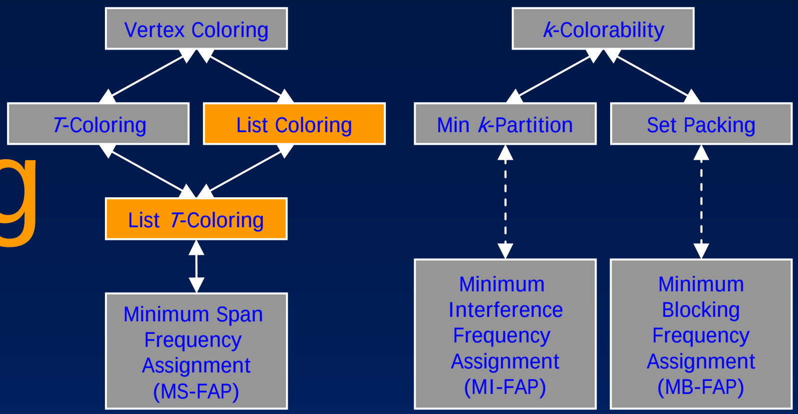
Colors: 3
Span: 4

Minimization of
number of colors and span
are not equivalent!

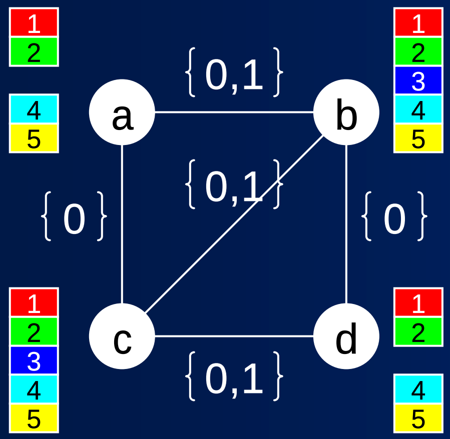


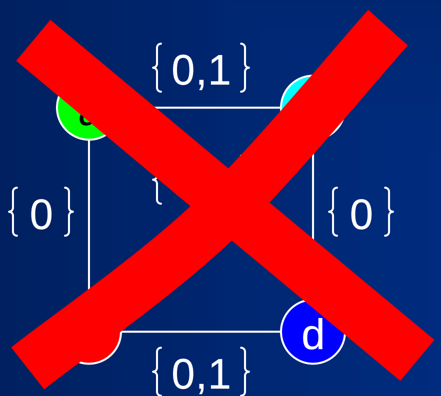
Colors: 4
Span: 3

FAP & List-*T*-Coloring

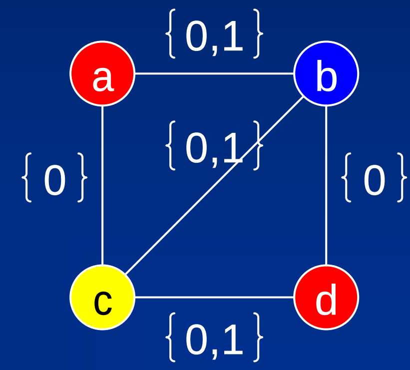


Locally blocked channels:
Sets of forbidden colors B_v



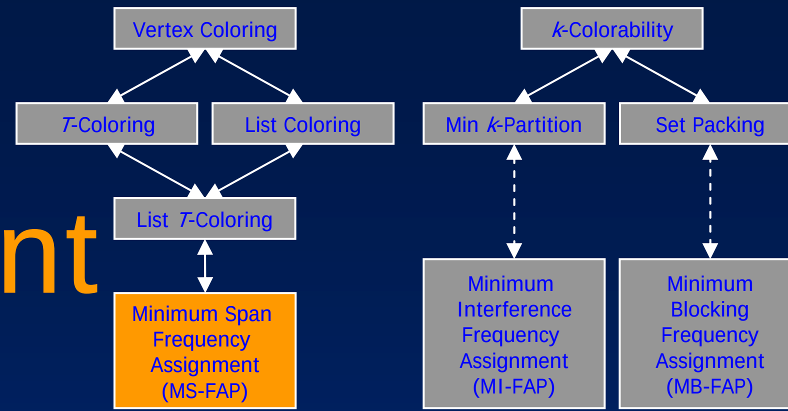


No solution with span 3 !



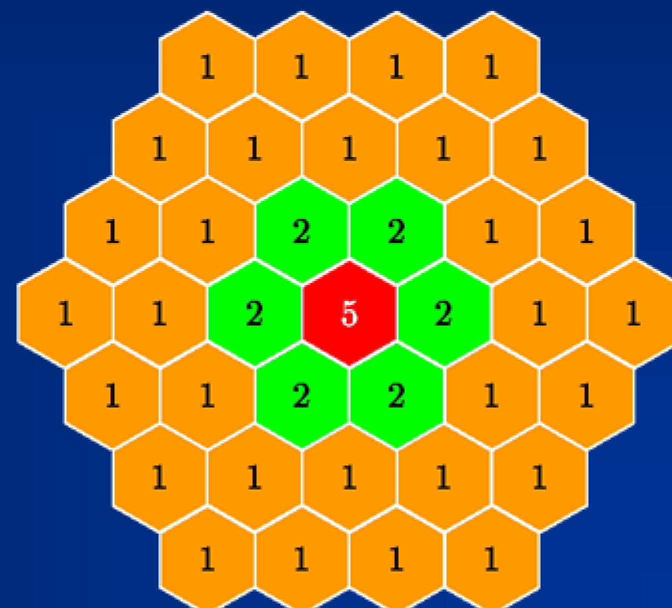
Colors: 3
Span: 4

Minimum Span Frequency Assignment



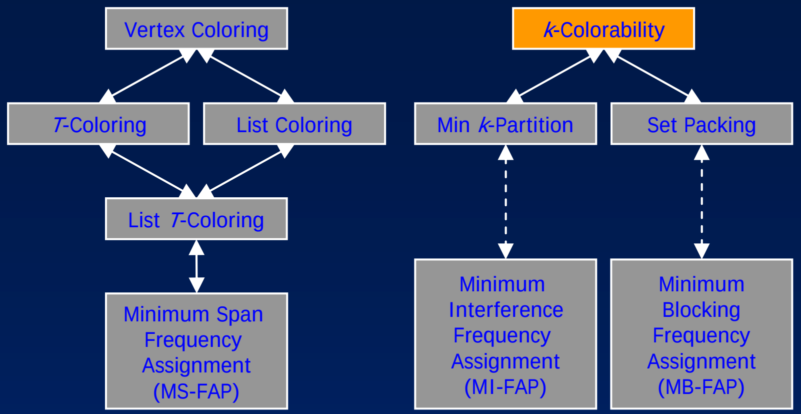
- List-T-Coloring (+ multiplicity)
- Benchmarks: Philadelphia instances

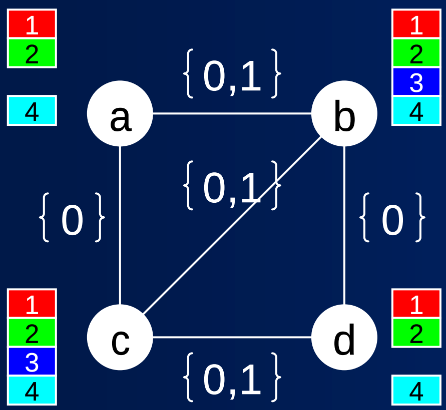
Channel requirements (P1)
Optimal span = 426



Separation distances

Fixed Spectrum





License for frequencies {1,...,4}

No solution with span 3

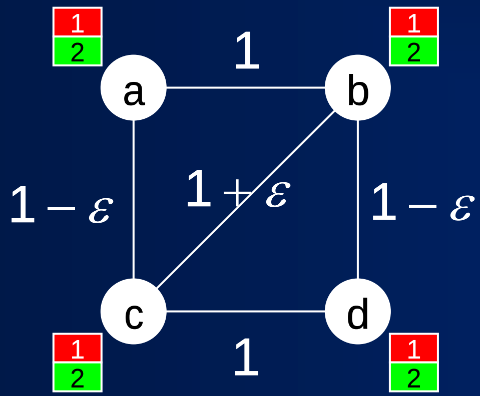
- Is the graph span-*k*-colorable?
- Complete assignment: minimize interference
- Partial assignment without interference

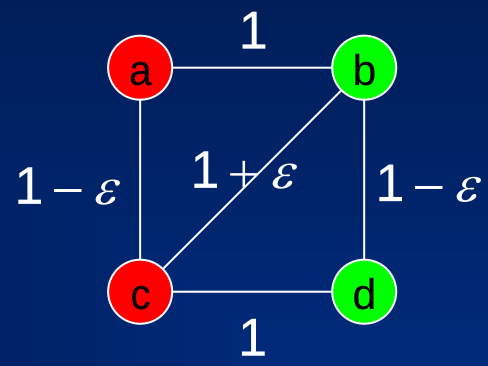
Hard & Soft constraints

- How to evaluate “infeasible” plans?
  - Hard constraints: separation, local blockings
  - Soft constraints: co- and adjacent-channel interference
- Measure of violation of soft constraints: penalty functions

$$p_{vw}(f,g) = \begin{cases} c^{co}(vw) & \text{if } f = g \\ c^{ad}(vw) & \text{if } |f-g| = 1 \\ 0 & \text{otherwise} \end{cases}$$

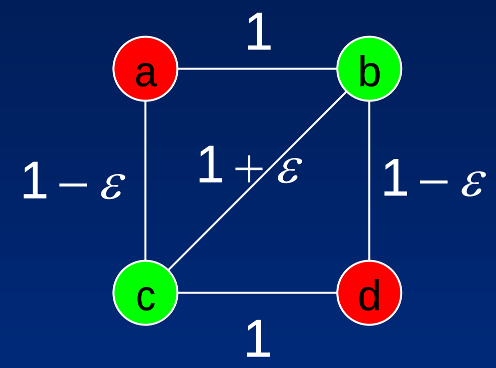
Evaluation of infeasible plans





Total penalty: $2-2\varepsilon$

Maximum penalty: $1-\varepsilon$



Total penalty: $1+\varepsilon$

Maximum penalty: $1+\varepsilon$

- Minimizing total interference
- Minimizing maximum interference
  – Use of threshold value, binary search

What is a good objective?

Keep interference information!

Use the available spectrum!

Minimize max interference

T-coloring (min span): Hale; Gamst; ...

Minimize sum over interference

Duque-Anton et al.; Plehn; Smith et al.; ...

Minimize max "antenna" interference

Fischetti et al.; Mannino, Sassano

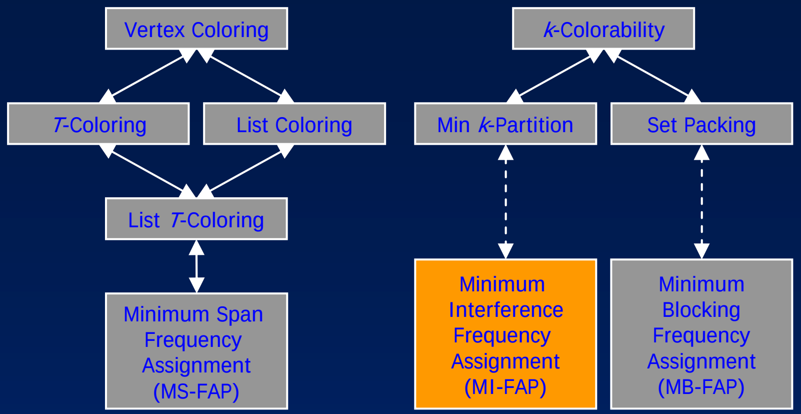
Our Model

Carrier Network:

$$N = (V, E, C, \{B_v\}_{v \in V}, d, c^{co}, c^{ad})$$

- (V,E) is an undirected graph
- C is an interval of integers (spectrum)
- $B_v \subseteq C$ for all $v \in V$ (blocked channels)
- $d : E \to Z_+$ (separation)
- $c^{co}, c^{ad} : E \to [0,1]$ (interference)

Minimum Interference Frequency Assignment



Integer Linear Program:

$$
\begin{aligned}
\min \quad & \sum_{vw \in E^{co}} c^{co}_{vw} z^{co}_{vw} + \sum_{vw \in E^{ad}} c^{ad}_{vw} z^{ad}_{vw} & \\
s.t. \quad & \sum_{f \in F_v} x_{vf} = 1 & \forall v \in V \\
& x_{vf} + x_{wg} \leq 1 & \forall vw \in E^{d}, \left| f - g \right| < d(vw) \\
& x_{vf} + x_{wf} \leq 1 + z^{co}_{vw} & \forall vw \in E^{co}, f \in F_v \cap F_w \\
& x_{vf} + x_{wg} \leq 1 + z^{ad}_{vw} & \forall vw \in E^{ad}, \left| f - g \right| = 1 \\
& x_{vf}, z^{co}_{vw}, z^{ad}_{vw} \in \{0,1\} & \forall v \in V, f \in C \setminus B_v, \forall vw \in E^{co}, \forall vw \in E^{ad}
\end{aligned}
$$

A Glance at some Instances

Instance	|V|	density [%]	minimum degree	average degree	maximum degree	diameter	clique number
k	267	56,8	2	151,0	238	3	69
B-0-E-20	1876	13,7	40	257,7	779	5	81
f	2786	4,5	3	135,0	453	12	69
h	4240	5,9	11	249,0	561	10	130

Expected graph properties: planarity,...

Computational Complexity

Neither high quality nor feasibility are generally achievable within practical running times:

- Testing for feasibility is NP-complete.
- There exists an $\varepsilon > 0$ such that FAP cannot be "approximated" within a factor of $|V|^{\varepsilon}$ unless $P = NP$.

Heuristic Solution Methods

- Greedy coloring algorithms,
- DSATUR,
- Improvement heuristics,
- Threshold Accepting,
- Simulated Annealing,
- Tabu Search,
- Variable Depth Search,
- Genetic Algorithms,
- Neural networks,
- etc.

Heuristics

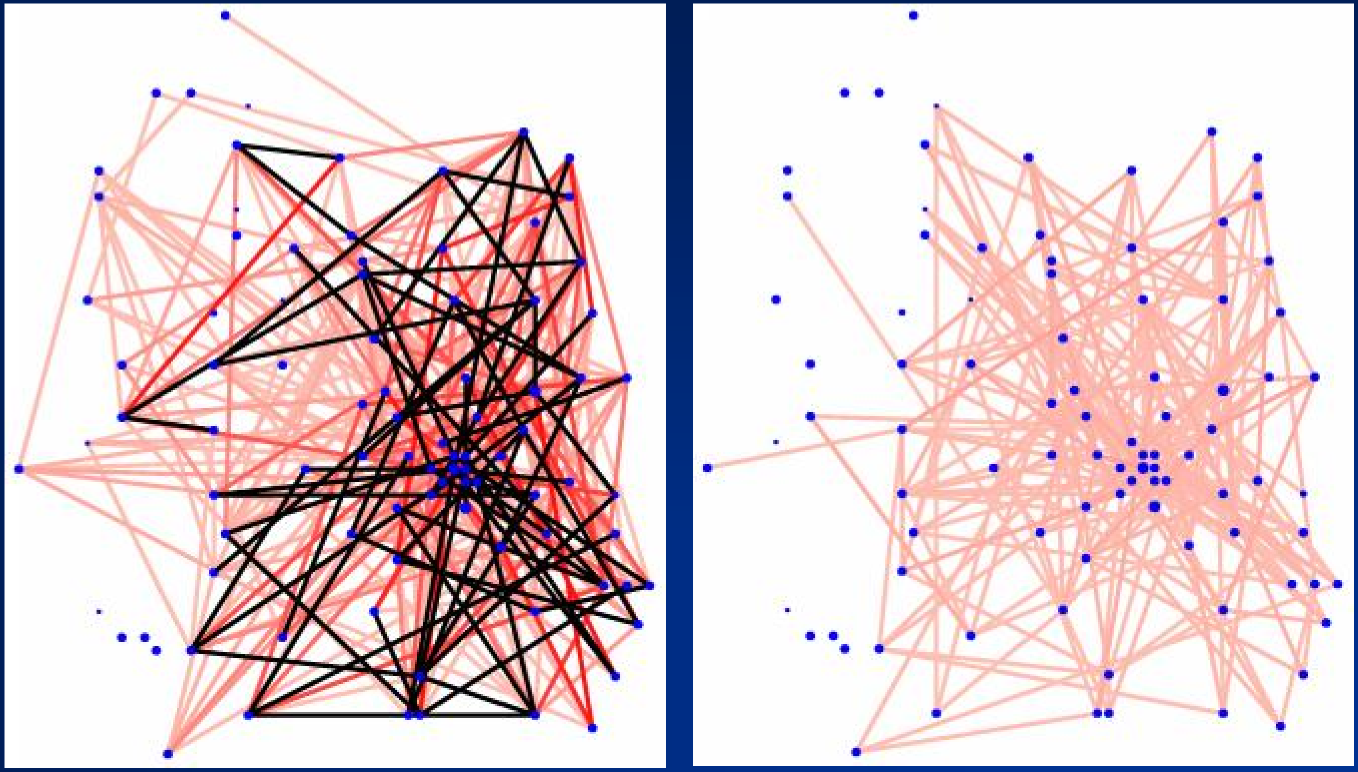
Heuristic	Rating	Category
T-coloring	o	construction heuristics
Dual Greedy	--	construction heuristics
DSATUR with Costs	++	construction heuristics
Iterated 1-Opt	o	(randomized) local search
Simulated Annealing	+	(randomized) local search
Tabu-Search	o	(randomized) local search
Variable Depth Search	++	(randomized) local search
MCF	-	other improvement heuristics
B&C-based	+	other improvement heuristics

Region with “Optimized Plan”

Instance k, a “toy case” from practice

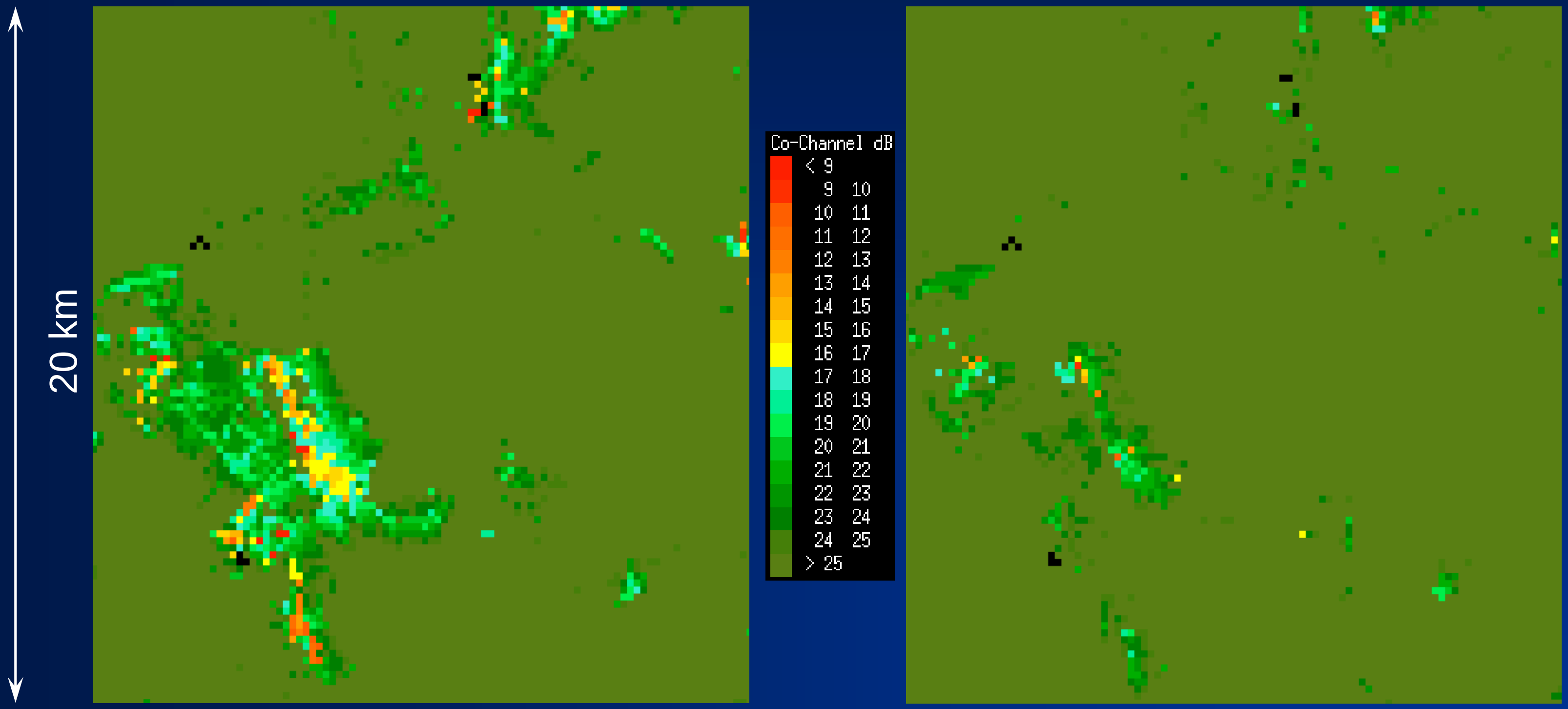
264 cells
267 TRXs
50 channels

57% density
151 avg.deg.
238 max.deg.
69 clique size



DC5-VDS: Reduction 96,3%

co-channel C/I worst Interferer



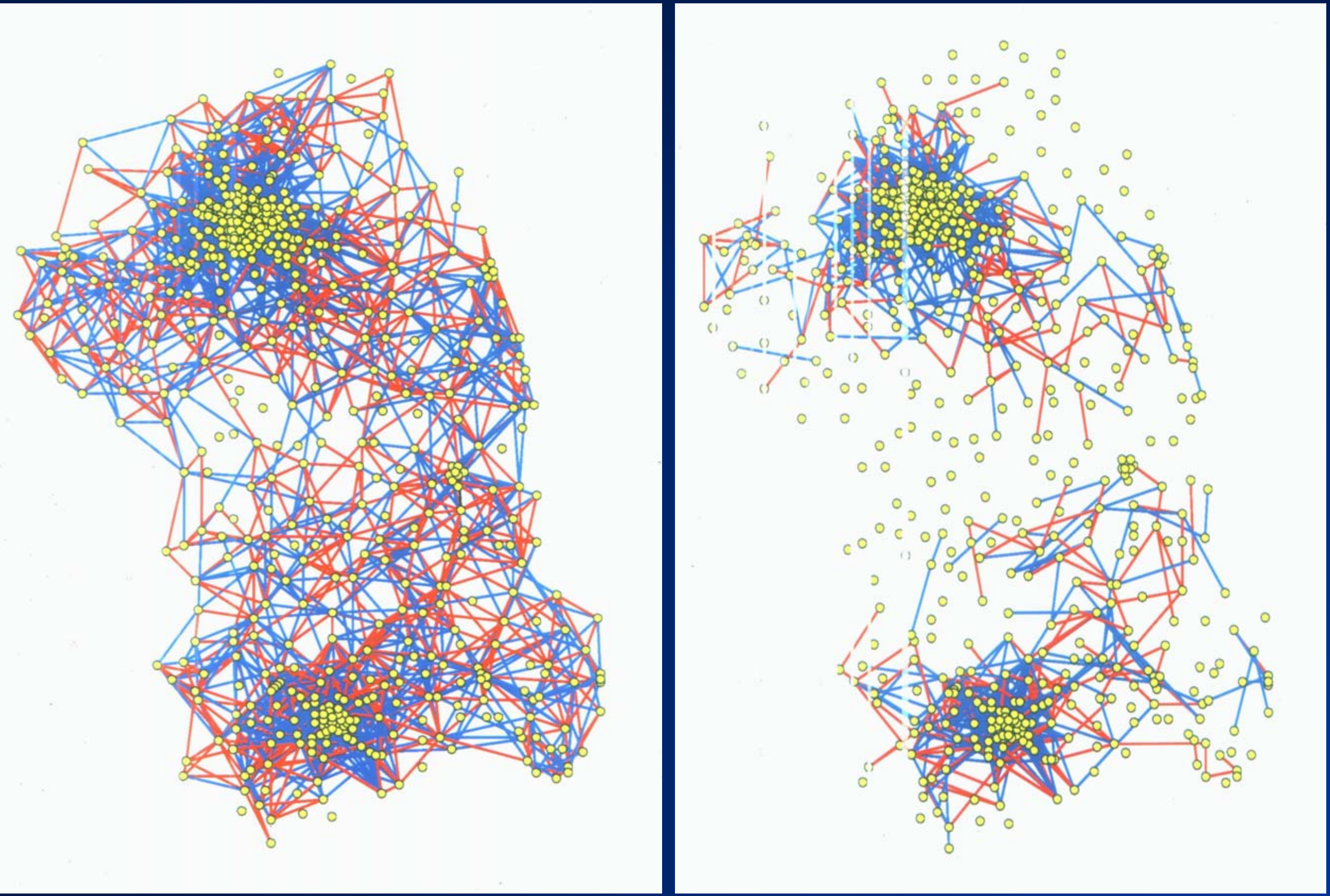
Mobile Systems International Plc.

Mobile Systems International Plc.

Commercial software

DC5-IM

Region Berlin - Dresden

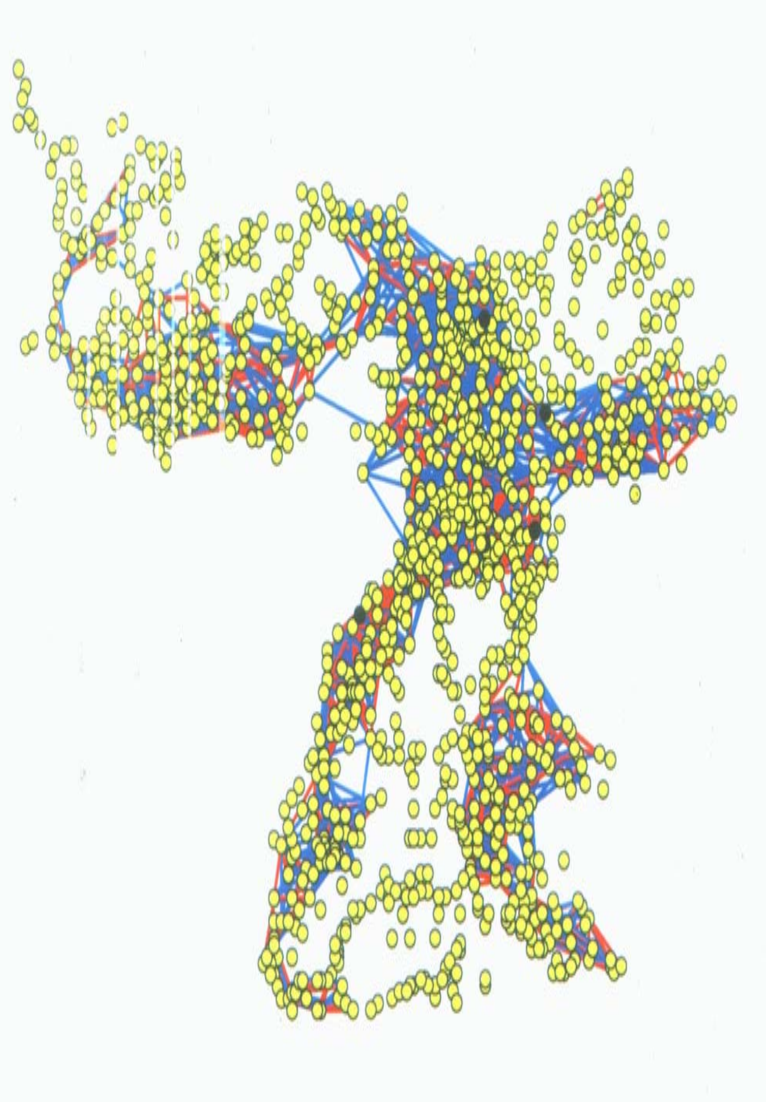


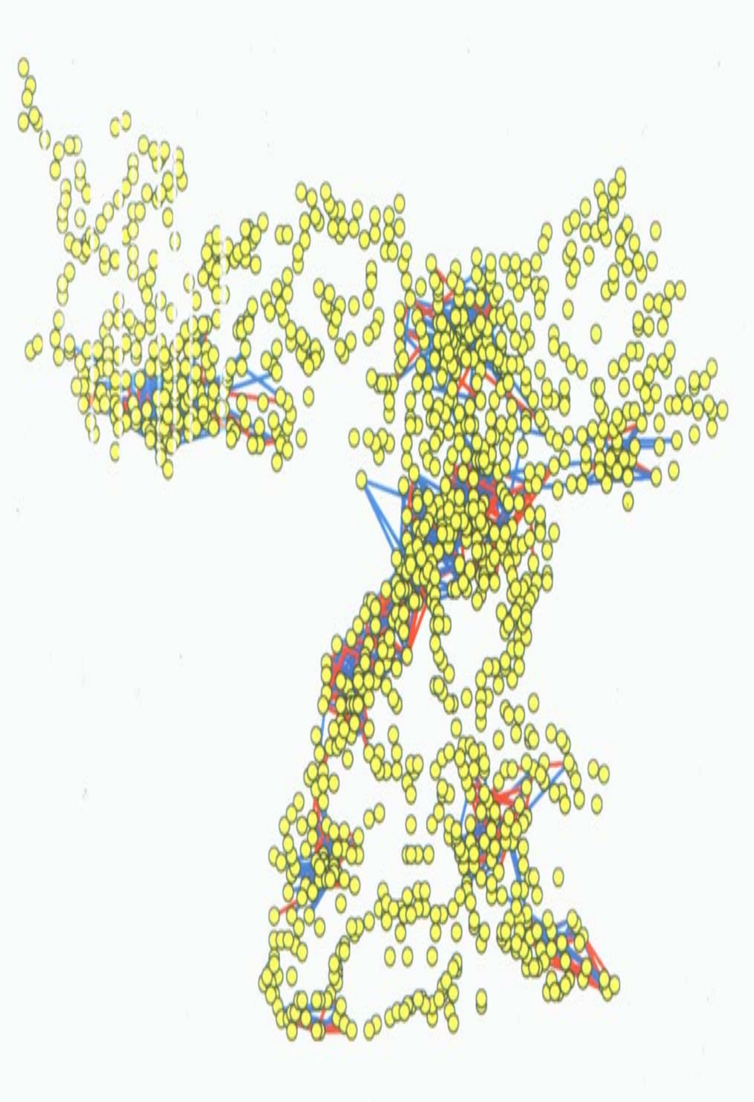
2877 carriers

50 channels

Interference reduction: 83.6%

Region Karlsruhe





2877
Carriers

75 channels

Interference
Reduction:
83.9 %

Guaranteed Quality

Optimal solutions are out of reach!

Enumeration: $50^{267} \approx 10^{197}$ combinations (for trivial instance k)

Hardness of approximation

Polyhedral investigation (IP formulation)

Aardal et al.; Koster et al.; Jaumard et al.; …

Used for adapting to local changes in the network

Lower bounds - study of relaxed problems

Lower Bounding Technology

- LP lower bound for coloring
- TSP lower bound for *T*-coloring
- LP lower bound for minimizing interference
- Tree Decomposition approach
- Semidefinite lower bound for minimizing interference

Region with “Optimized Plan”

Instance k, the “toy case” from practice

264 cells

267 TRXs

50 channels

57% density

151 avg.deg.

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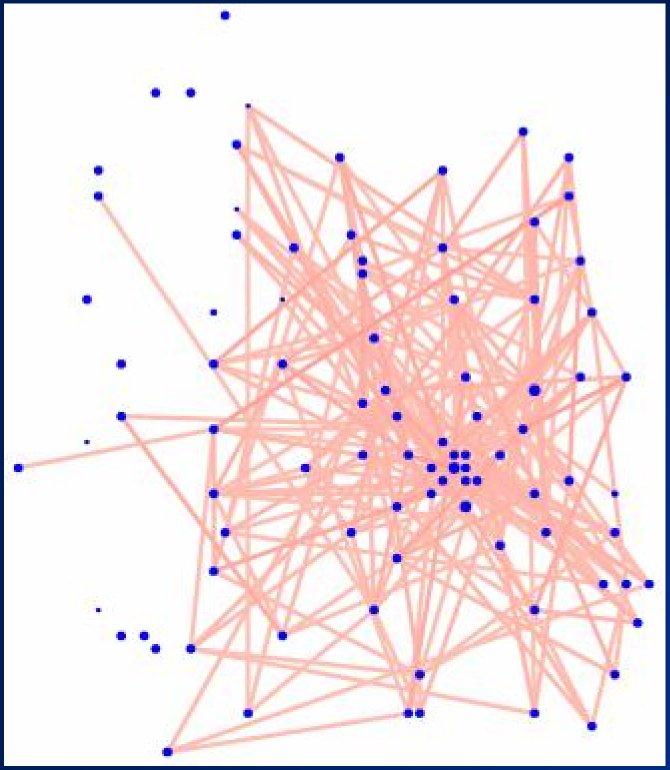
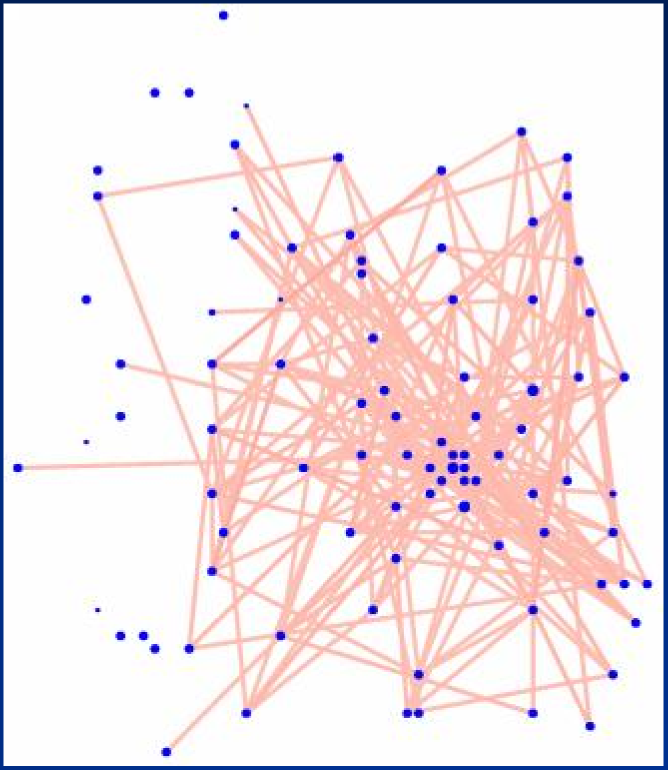


DC5-VDS



Further Reduction: 46.3%

A Simplification of our Model

Simplified Carrier Network:

$$N = (V, E, C, \{B_v\}_{v \in V}, d, c^{co}, c^{ad})$$

- (V,E) is an undirected graph
- C is an interval of integers (spectrum)
- $B_v \subseteq C$ for all $v \in V$ (blocked channels)
- $d : E \to Z_+$ {0, 1} (separation)
- $c^{co}, c^{ad} : E \to [0,1]$ (interference)

MIN k-Partition

- No blocked channels
- No separation constraints larger than one
- No adjacent-channel interference

- Invariant under permutation of channels!

- **min k-partition (max k-cut)**
  Chopra & Rao; Deza et al.; Karger et al.; Frieze & Jerrum
- IP, LP-based B&C, SDP

MIN k-Partition

Given: an undirected graph $G = (V,E)$ together with real edge weights w_{ij} and an integer k.

Find a partition of the vertex set into (at most) k sets $V_1, \ldots, V_k$ such that the sum of the edge weights in the induced subgraphs is minimal!

$$\min_{\substack{V_1,\ldots,V_k \\ \text{partition of } V}} \sum_{p=1}^{k} \sum_{i,j \in V_p} w_{ij}$$

NP-hard to approximate optimal solution value.

Integer Linear Programmming

(ILP)

$$\min \sum_{i,j \in V} w_{ij} z_{ij}$$

$$z_{ih} + z_{hj} - z_{ij} \leq 1 \qquad \forall h, i, j \in V$$ -> partition consistent

$$\sum_{i,j \in Q} z_{ij} \geq 1 \qquad \forall\, Q \subseteq V \text{ with } |Q| = k+1$$

-> use at most k blocks

$$z_{ij} \in \{0, 1\}$$

Number of ILP inequalities (facets)

Instance*	|V|	k	Triangle	Clique Inequalities
cell.k	**69**	**50**	**157182**	**17231414395464984**
B-0-E	81	75	255960	25621596
B-1-E	**84**	**75**	**285852**	**43595145594**
B-2-E	93	75	389298	1724861095493098563
B-4-E	120	75	842520	1334655509331585084721199905599180
B-10-E	174	75	2588772	361499854695979558347628887341189586948364637617230

Vector Labeling

Lemma: For each k, n ($2 \leq k \leq n+1$) there exist k unit vectors $u_1, \ldots, u_k$ in n-space, such that their mutual scalar product is $-1/(k-1)$. (This value is least possible.)

Fix $U = \{u_1, \ldots, u_k\}$ with the above property, then the min k-partition problem is equivalent to:

$$\min_{\substack{\phi: V \to U \\ i \mapsto \phi_i}} \sum_{ij \in E} \left(\frac{k-1}{k} \langle \phi_i, \phi_j \rangle + \frac{1}{k} \right) w_{ij}$$

$X = [\langle \phi_i, \phi_j \rangle]$ is positive semidefinite, has 1's on the diagonal, and the rest is either $-1/(k-1)$ or 1.

Semidefinite Relaxation

(SDP)

$$\min \sum_{ij \in E(K_n)} w_{ij} \frac{(k-1)\,V_{ij}+1}{k}$$

$$V_{ii} = 1 \qquad \forall i \in V$$

$$V_{ij} \geq \frac{-1}{k-1} \qquad \forall i,j \in V$$

$$V \succeq 0$$

Solvable in polynomial time!

Given V, let $z_{ij} := ((k\text{-}1)\ V_{ij} + 1)/k$, then:

- z_{ij} in [0,1]
- $z_{ih} + z_{ih} - z_{ij} < \sqrt{2}$ $(<=1)$
- $\sum_{i,j \text{ in } Q} z_{ij} > ½$ $(>=1)$

(SDP) is an approximation of (ILP)

Computational Results

S. Burer, R.D.C Monteiro, Y. Zhang; Ch. Helmberg; J. Sturm

Instance	clique cover	min k-part.	*heuristic*	clique cover	min k-part.	*heuristic*
cell.k	**0,0206**	**0,0206**	***0,0211***	**0,0248**	**0,1735**	**0,4023**
B-0-E	0,0016	0,0013	*0,0016*	0,0018	0,0096	*0,8000*
B-1-E	**0,0063**	**0,0053**	***0,0064***	**0,0063**	**0,0297**	**0,8600**
B-2-E	0,0290	0,0213	*0,0242*	0,0378	0,4638	*3,1700*
B-4-E	0,0932	0,2893	*0,3481*	0,2640	4,3415	*17,7300*
B-10-E	0,2195	2,7503	*3,2985*			*146,2000*
	maximal clique			entire scenario		

Lower bound on co-channel interference by a factor of 2 to 85 below co- and adjacent-channel interference of best known assignment.

Semidefinite Conclusions

Lower bounding via
Semidefinite Programming works (somewhat),
at least better than LP!

- Challenging computational problems
- Lower bounds too far from cost of solutions to give strong quality guarantees
- How to produce good k-partitions starting from SDP solutions?

Literature (ZIB PaperWeb)

K. Aardal, S. van Hoesel, A. Koster, C. Mannino, A. Sassano, “Models and Solution Techniques for the Frequency Assignment Problem”, ZIB-report 01-40, 2001.

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A. Eisenblätter, M. Grötschel, A. Koster, “Frequency Planning and Ramifications of Coloring”, Discussiones Mathematicae, Graph Theory, 22 (2002) 51-88.

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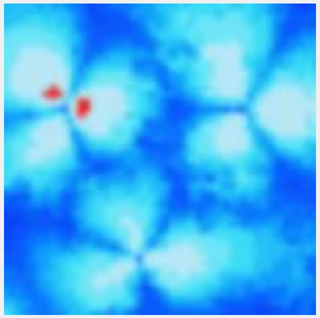
FAP web – A website devoted to Frequency Assignment:

http://fap.zib.de

Almost everything you want to know about Frequency Assignment: FAP web



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- collections of test problems for benchmarking solution methods,
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