LDPC

Communication Project

Implementation and Analysis of LDPC codes over BEC

Bar-Ilan university, school of engineering
Chen Koker and Maytal Toledano

October

Outline

- Definitions of Channel and Codes.
- Introduction to LDPC.
- Decoding base on Belief Propagation.
- Implementation of the decoder.
- Density Evolution.
- Results.

Project Description

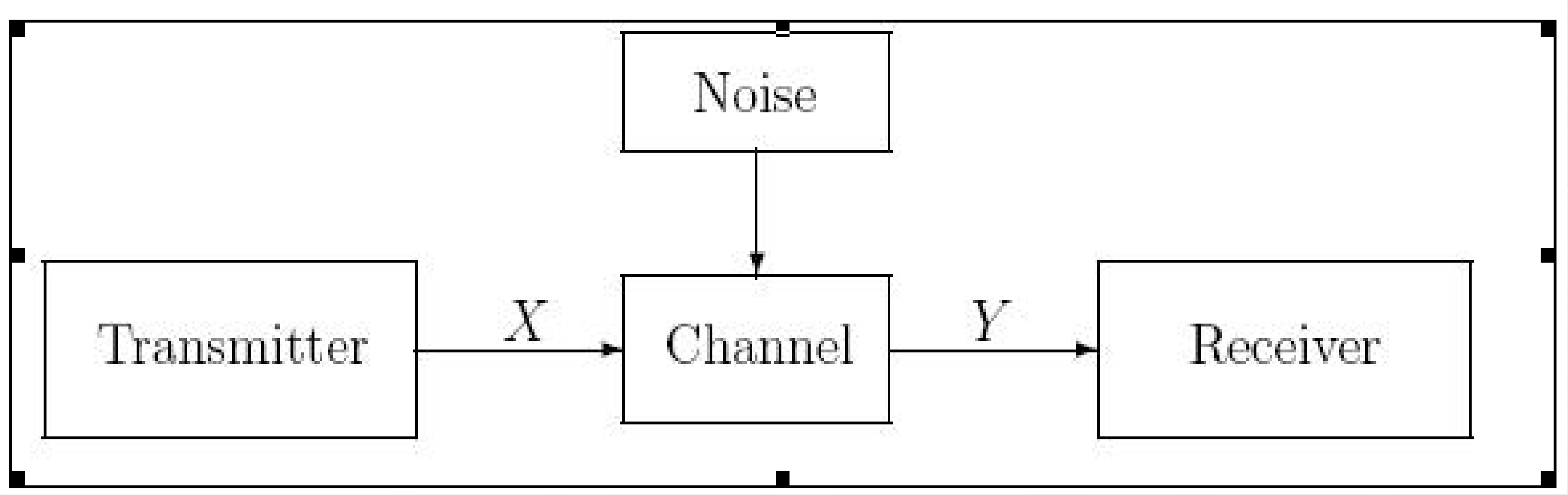
- Implementation decoder of LDPC codes over BEC.
- Density Evolution algorithm.
- Asymptotic analytic results versus simulations results.

Outline

- **Definitions of Channel and Codes.**
- Introduction to LDPC.
- Decoding base on Belief Propagation.
- Implementation of the decoder.
- Density Evolution.
- Results.

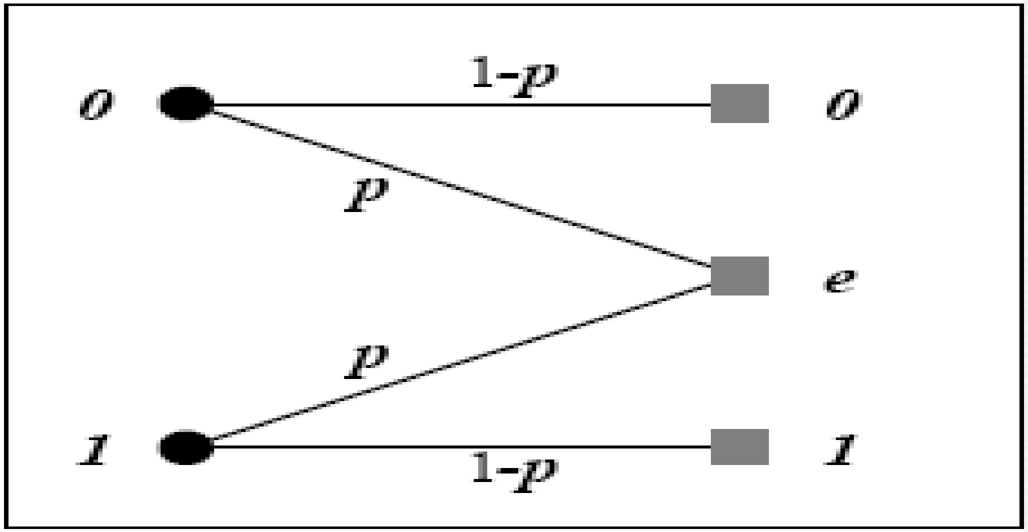
Definitions of channel

- **Digital channel:**



Definitions of channel

- **BEC = Binary Erasure Channel:**



"E" ➔ equal probability for "0" and "1“.

Definitions of channel

LDPC

- **BEC = Binary Erasure Channel:**

we can assume that the conditional probabilities are

$$p(x_i = 1 \mid y_i = 1) = 1 \qquad p(x_i = 1 \mid y_i = 0) = 0$$

$$p(x_i = 0 \mid y_i = 0) = 1 \qquad p(x_i = 0 \mid y_i = 1) = 0$$

$$p(x_i = 0 \mid y_i = E) = 0.5 \qquad p(x_i = 0 \mid y_i = E) = 0.5$$

Definitions of channel

- **C = Channel Capacity:**

  The amount of information that can be reliably transmitted over a channel.

  In **BEC** the Capacity is:

$$C = 1 - Pe$$

Pe = probability of erasure.

- **Block code:**

$$U_{1 \times k} \xrightarrow{\text{channel encoding}} X_{1 \times n}$$

Definitions of codes

- **R = Code Rate:**

$$R = \frac{k}{n}$$

k - number of bits of the effective data.

n - number of bits of the effective data

+ redundancy bits.

Definitions of codes

- **G = Generator Matrix:**

$$G_{4\times 2} \cdot s^{T}{}_{2\times 1} = c_{4\times 1} \xrightarrow{\text{Channel}} r_{4\times 1}$$

$$\begin{pmatrix} 1 & 1 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{pmatrix} \bullet \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \end{pmatrix} \xrightarrow{\text{BEC Channel}} \begin{pmatrix} 0 \\ E \\ 1 \\ 1 \end{pmatrix}$$

G

Definitions of codes

- ✓ **Notice H*G = 0**

  (H = Parity Matrix)

$$H_{2\times 4} \cdot G_{4\times 2} = \bar{0}_{2\times 2}$$

$$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \end{pmatrix} \bullet \begin{pmatrix} 1\,1 \\ 0\,0 \\ 1\,0 \\ 0\,1 \end{pmatrix} = \bar{0}$$

Definitions of codes

- **H = Parity Check Matrix:**

$$H_{2\times4}\cdot r_{4\times1}=\bar{0}$$

Codeword pass BEC

H

$$\begin{pmatrix}0&1&0&0\\1&0&1&1\end{pmatrix}\bullet\begin{pmatrix}0\\E\\1\\1\end{pmatrix}=\left(\bar{0}\right)$$

Outline

- Definitions of Channel and Codes.
- **Introduction to LDPC**
- Decoding base on Belief Propagation.
- Implementation of the decoder.
- Density Evolution.
- Results.

Introduction to LDPC

- Low density parity check code –
  the parity check matrix H is binary and sparse.

$$\begin{pmatrix} 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Introduction to LDPC

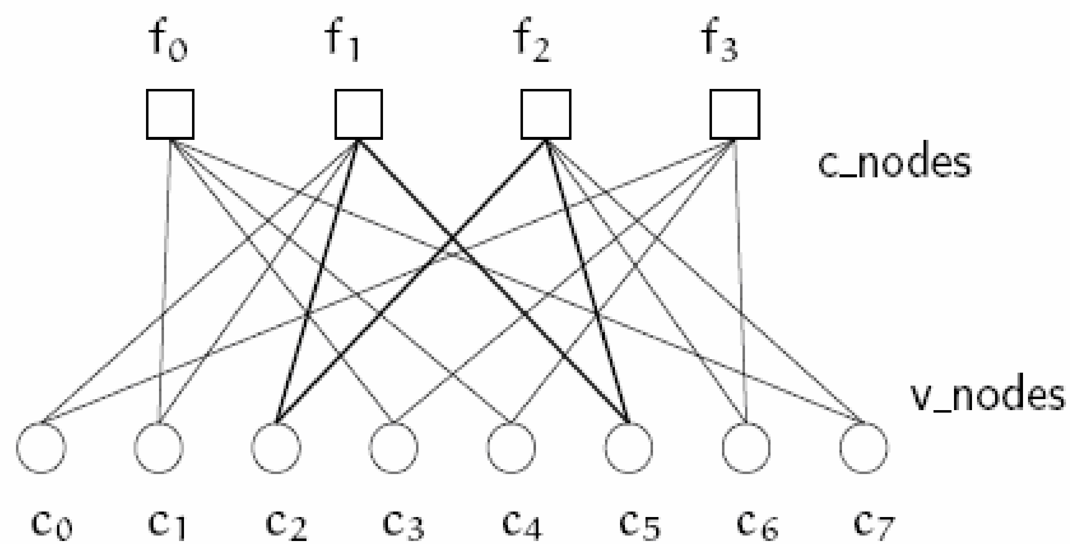
- LDPC codes represented by:

matrix

$$H = \begin{bmatrix} 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \end{bmatrix}$$

or by

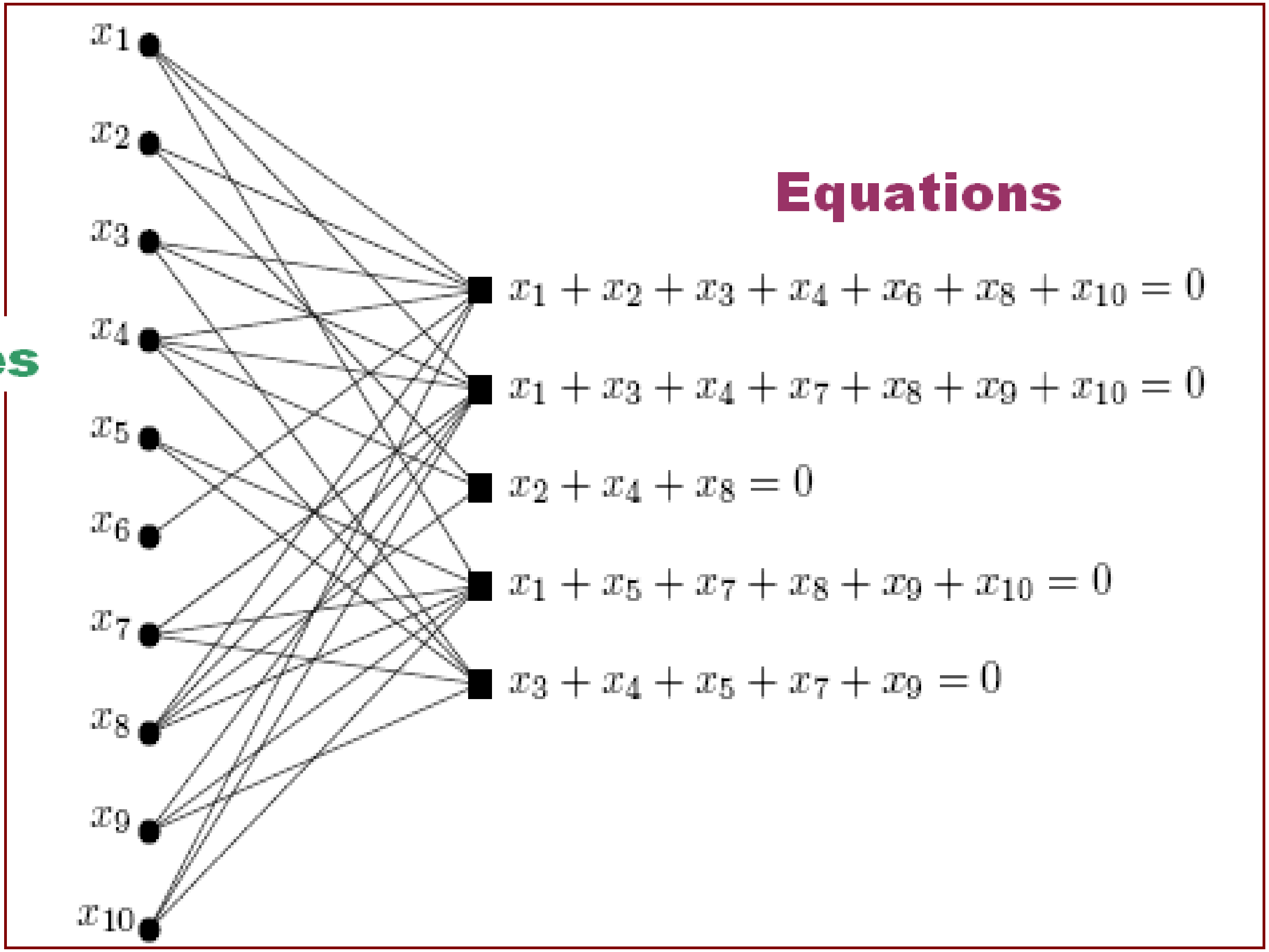
bipartite graph



Introduction to LDPC

- ***Tanner Graph – (bipartite graph)***

Variables



Outline

- Definitions of Channel and Codes.
- Introduction to LDPC.
- **Decoding base on Belief Propagation**
- Implementation of the decoder.
- Density Evolution.
- Results.

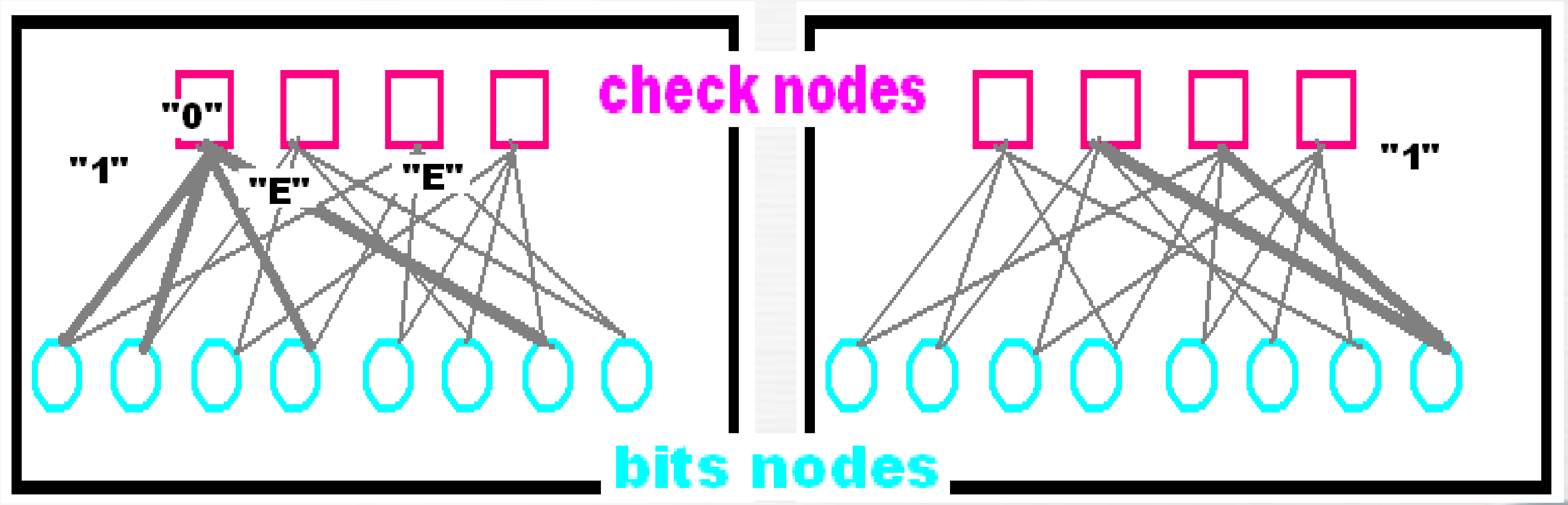
Decoding base on Belief Propagation

LDPC

- Decoding is **iterative algorithms** based on **message passing**.

Messages are passed from check nodes to bit nodes.

Messages are passed from bit nodes to check nodes.



LDPC BEC using Belief Propagation

The Iterative Algorithm:

- ***Step 1*** -

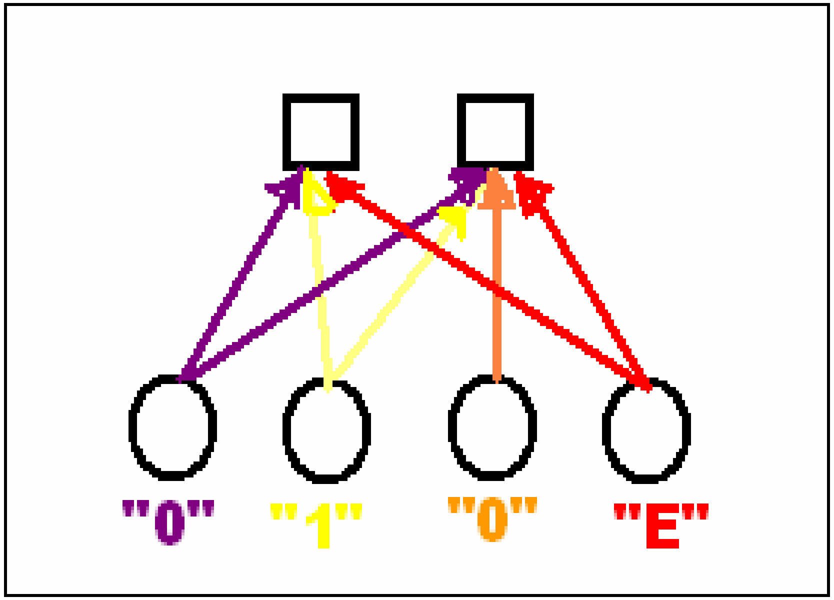
  All variable nodes send their qij messages.

$$q_{ij}(1) = P_i, \quad q_{ij}(0) = 1 - P_i$$

➔ ***Over BEC***

for "0" or "1" - $$q_{ij}(b) = 1,0$$

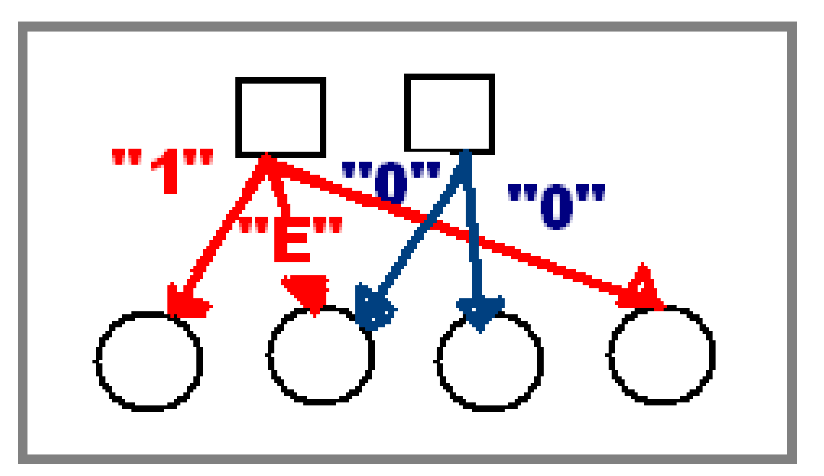
for "E" - $$q_{ij}(b) = 1/2$$



LDPC BEC using Belief Propagation

The Iterative Algorithm….

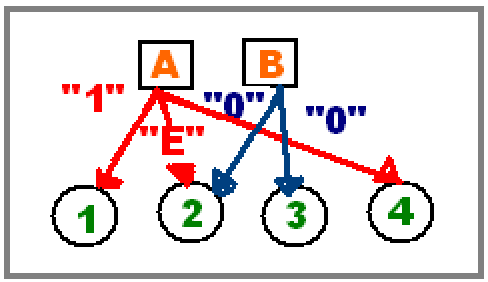
- ***Step 2*** -
  The check nodes calculate their response messages rji,



$$r_{ji}(0) = \frac{1}{2} + \frac{1}{2} \prod_{i' \in V_j \backslash i} (1 - 2q_{i'j}(1))$$

$$r_{ji}(1) = 1 - r_{ji}(0)$$

LDPC BEC using Belief Propagation



The Iterative Algorithm….

- ***step 2….***
  For ***BEC*** it is ***Hard-Decision,***

  for "0" or "1" - **$r_{ji}(b) = 1,0$**

  for "E" - **$r_{ji}(b) = 1/2$**

→ ***In words, if all other bits in the equation are known the message is the correct value of the bit otherwise the message is 'E'.***

LDPC BEC using Belief Propagation

The Iterative Algorithm….

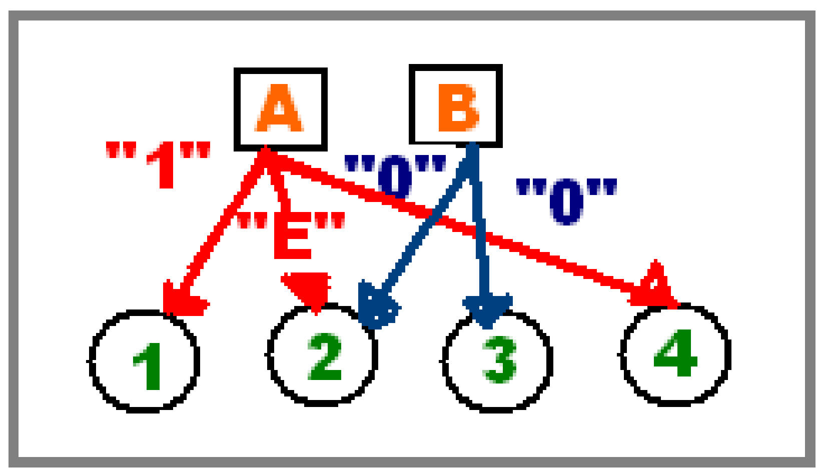
- *step 2….*

  The calculate is done by the logic XOR operation.

Equation A: X1 + X2 + X4 = 0

Equation A: 1 + E + 0 = 0

X2 → 1



Equation B: X2 + X3 = 0

Equation B: 0 + 0 = 0

LDPC BEC using Belief Propagation

The Iterative Algorithm….

- **Step 3** -

  The variable nodes update their response messages to the check nodes.

$$q_{ij}(0) = K_{ij}\,(1 - P_i) \prod_{j' \in C_i \backslash j} r_{j'i}(0)$$

$$q_{ij}(1) = K_{ij} P_i \prod_{j' \in C_i \backslash j} r_{j'i}(1)$$

LDPC BEC using Belief Propagation

The Iterative Algorithm….

- ***Step 3*** -

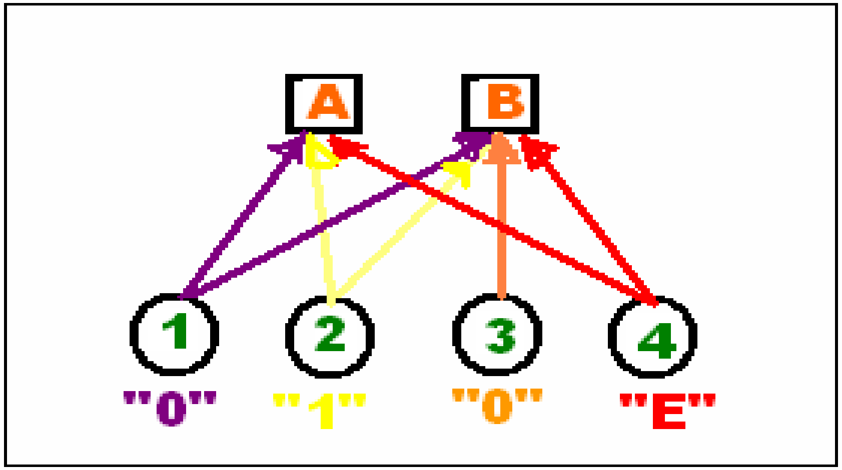
  Over BEC $q_{ij}(b) = 1, 0, 1/2$

  If the bit is already revealed then it sends it real value, otherwise it sends ‘E’.

LDPC BEC using Belief Propagation

The Iterative Algorithm….

Before at step 1:



At step 2 we found:

Equation A: X1 + X2 + X4 = 0

Equation A: 0 + 1 + E = 0

X4 → 1

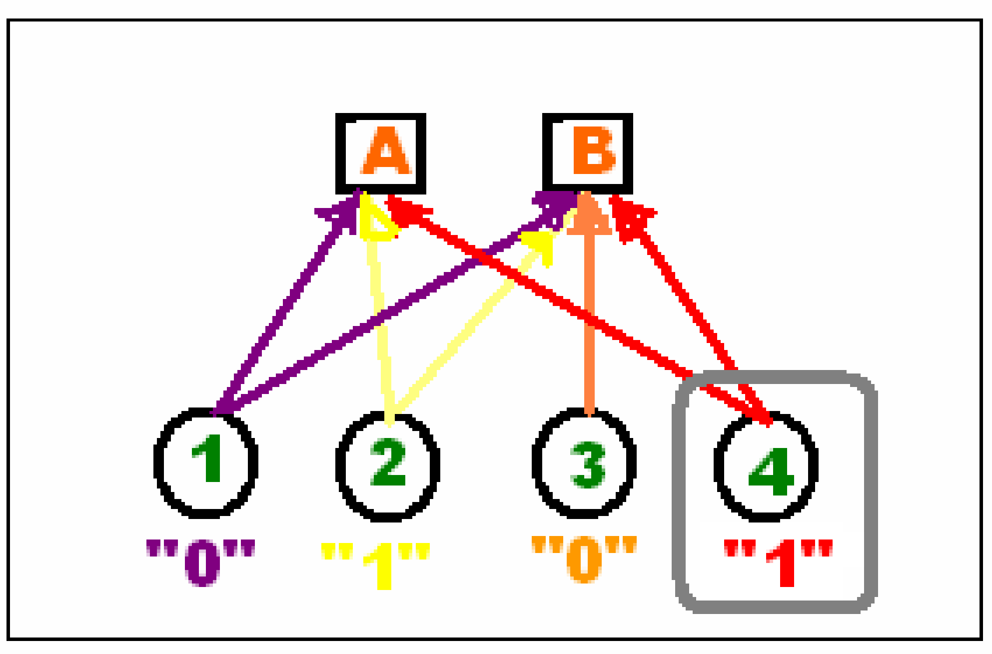
Equation B: X1+ X2 + X3 +X4 = 0

Equation B: 0 + 1 + 0 + E = 0

LDPC BEC using Belief Propagation

The Iterative Algorithm….

Now at step 3:

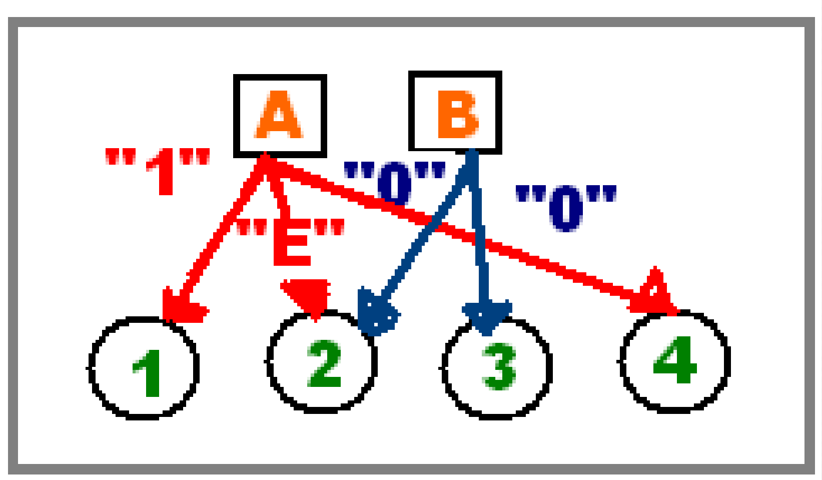


LDPC BEC using Belief Propagation

The Iterative Algorithm….

- ***Step 4*** -

  Go to step 2 (The check nodes calculate their response messages rji).



Algorithm Performance

- The algorithm for may be executed for a maximum number of rounds till:

1. It founds legal codeword→ not necessarily the right one.

2. It doesn't convergence to solution.

Algorithm Performance

- For **BEC** :

  1. It fulfils the **precise codeword**.

  2. It reach saturation -no solution.
     "Stopping Set" situation.

Algorithm Performance

The “Stooping Set” situation….

- The bits can not be decoded.

$$x_1 = E, x_2 = E, x_3 = E$$

$$c_1 = x_1 + x_2 + x_3 = 0$$

$$c_2 = x_1 + x_2 = 0$$

$$c_2 = x_1 + x_3 = 0$$

Outline

- Definitions of Channel and Codes.
- Introduction to LDPC.
- Decoding base on Belief Propagation.
- **Implementation of the decoder**
- Density Evolution.
- Results.

Implementation of the Decoder

- The implementation includes 3 components:

1. ***Initialization.***

2. ***Iterative Decoding.***

3. ***Analysis performance.***

Implementation of the Decoder

Iterative Decoding – efficient data structure

- ***Bit Vector*** –
  1. Value –'0','1', or 'E‘.
  2. Pointer to the equations it take part.
  3. Number of equations.

- ***Equation Vector*** –
  1. Number of known & unknown bits.
  2. Equation value of XOR function.

- ***Stack*** – index of unknown bits.

Implementation of the Decoder

Iterative Decoding

- The **stack** is scanned,

  Is update of unknown bit is possible?

  Yes: value updates in all the equations it involved.

  No: skip to the next unknown bit.

Implementation of the Decoder

Iterative Decoding

- The process relies on - equation with **only one unknown bit.**

- ***The Stack halt condition –***

1. The stack is empty.
2. No bit was updated *("Stopping Set").*

Outline

- Definitions of Channel and Codes.
- Introduction to LDPC.
- Decoding base on Belief Propagation.
- Implementation of the decoder.
- **Density Evolution**
- Results.

Density Evolution

- **DE = Density Evolution:**

An asymptotic analysis method for LDPC code performance under the Message-passing decoding.

Density Evolution

- f_t **= probability of bit to be unknown**
  after t iterations of massage passing algorithm.

$$f_t = P_e(1-(1-f_{t-1})^{(n-1)})^{(m-1)}$$

p_e – initial probability of error.

n – number of bits in each equation.

m – number of equations each bit involved.

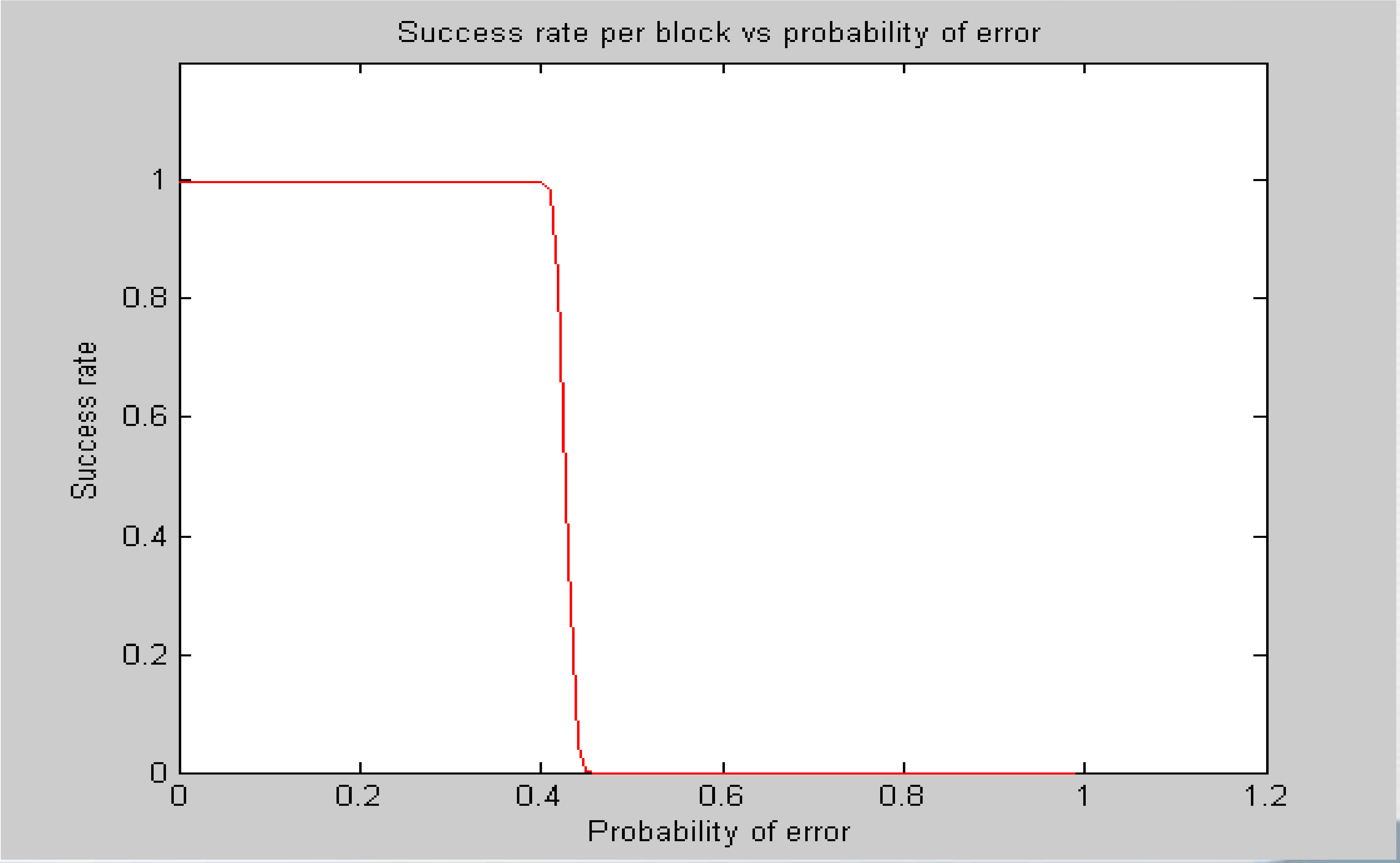
Outline

- Definitions of Channel and Codes.
- Introduction to LDPC.
- Decoding base on Belief Propagation.
- Implementation of the decoder.
- Density Evolution.
- **Results**

Results

LDPC

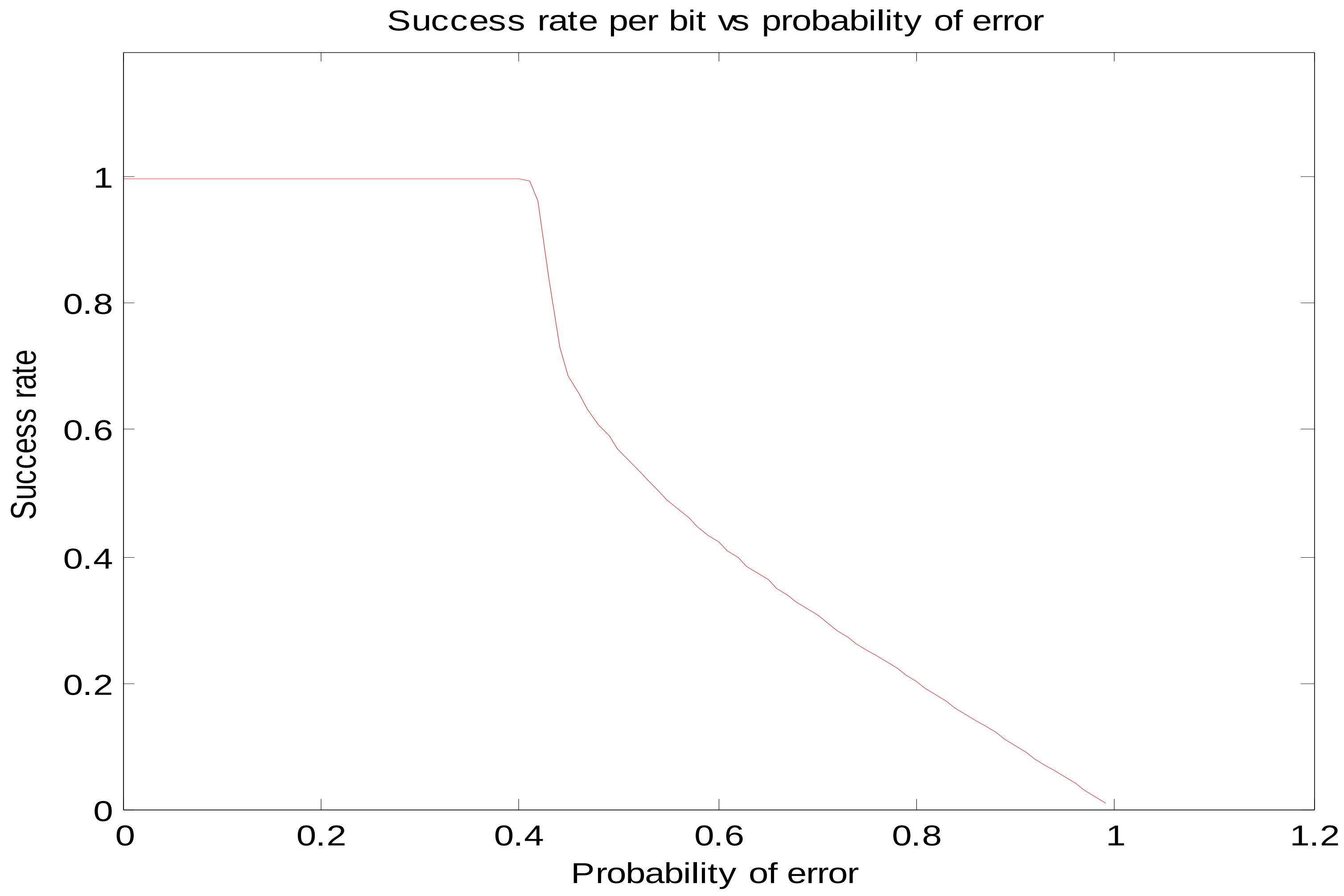
- ***"waterfall"*** - success per 5000 bit.



Results

LDPC

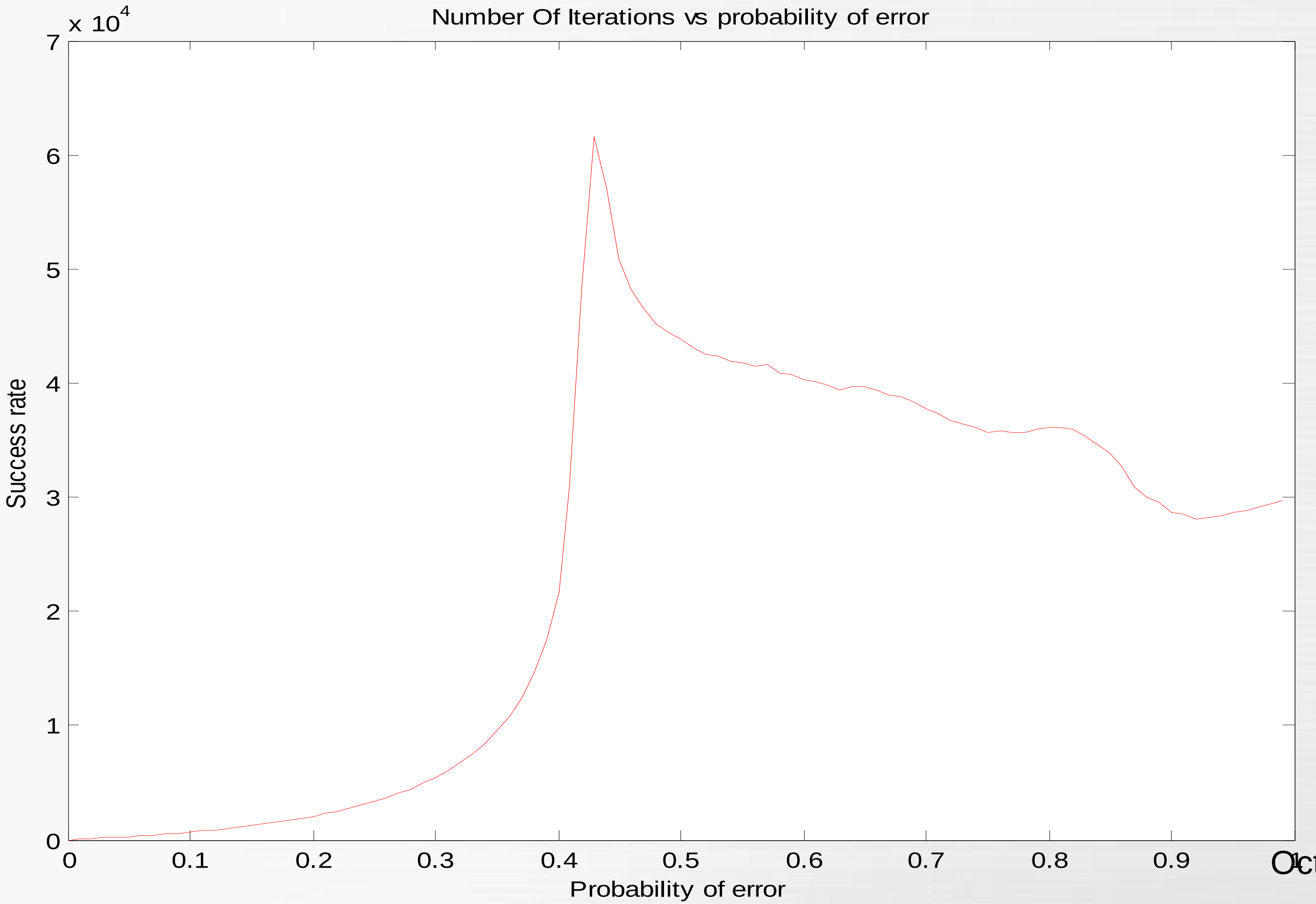
- ***"waterfall"*** - success per bit.



Results

LDPC

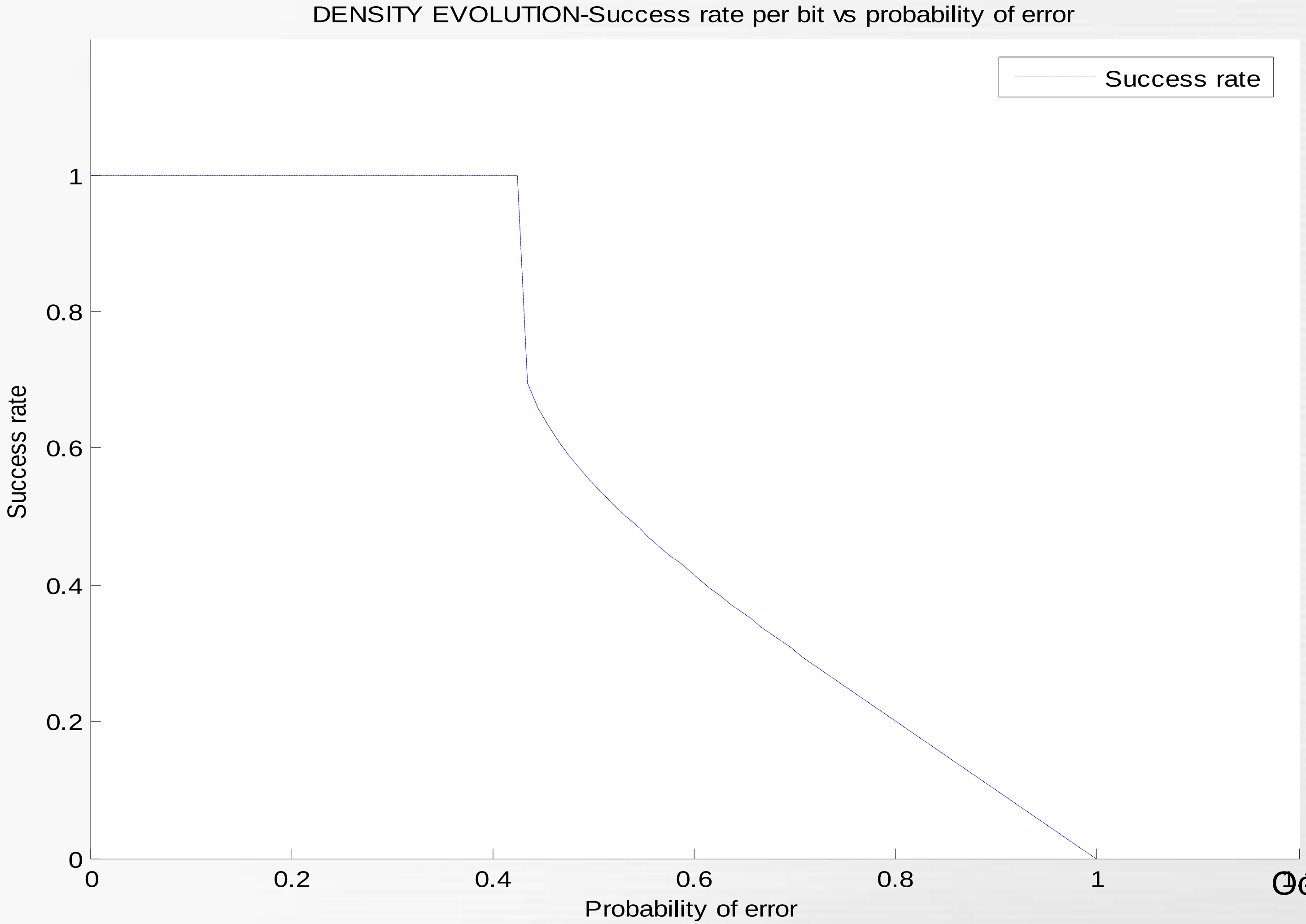
- **Number of iteration to reach a decision:**



Results

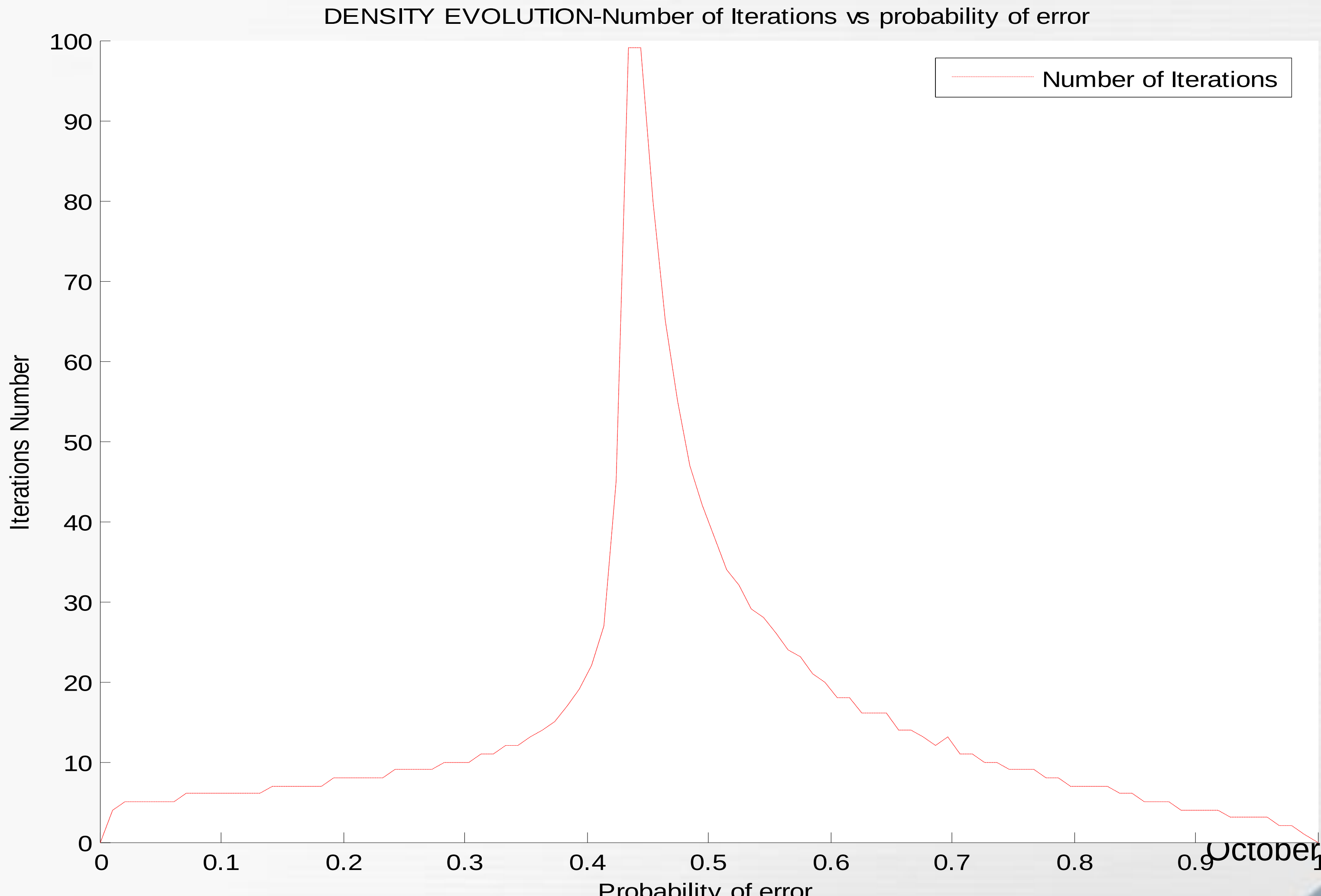
LDPC

- **Density Evolution:**



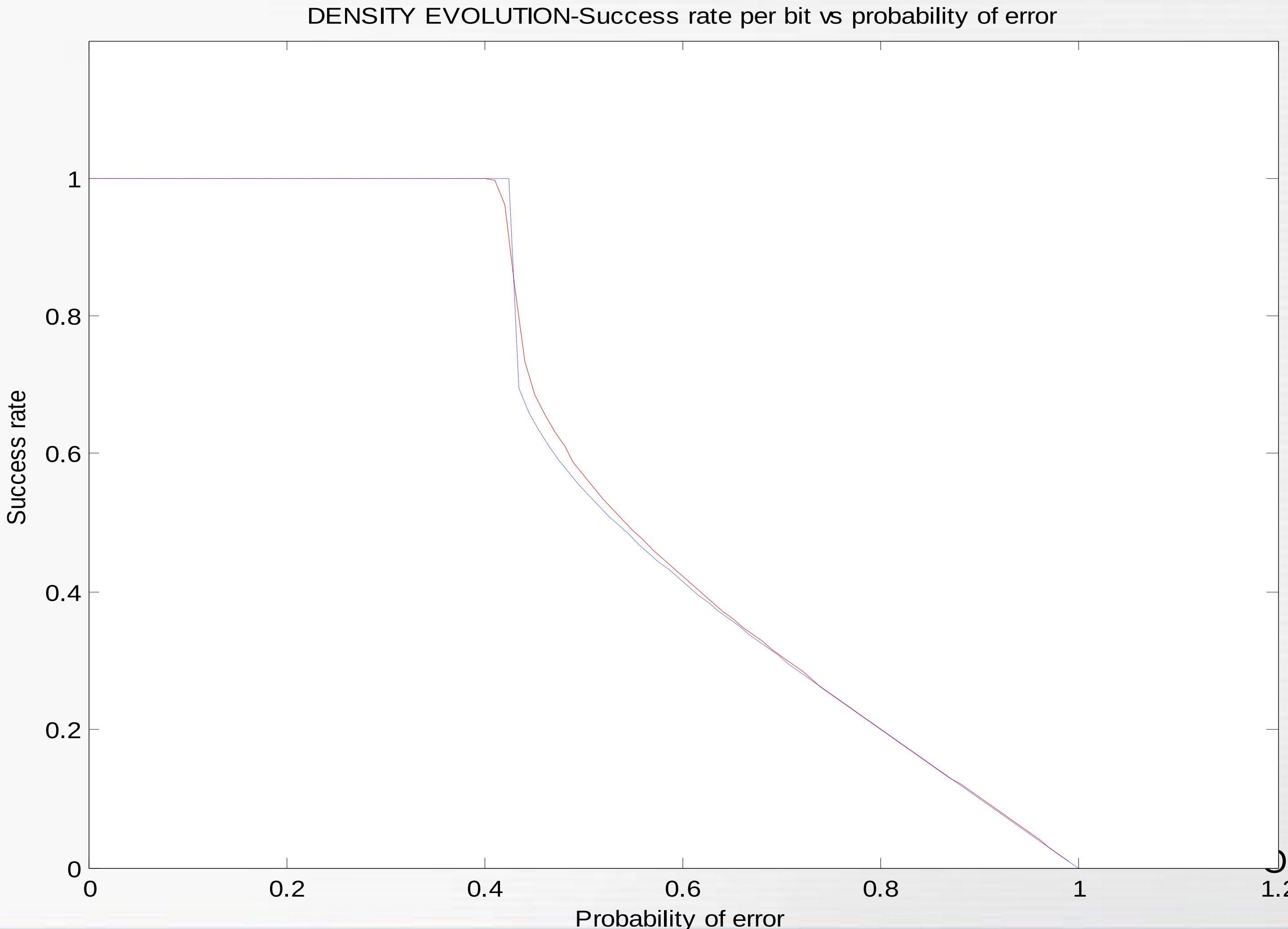
Results

- **Density Evolution:** *Number of iteration to reach a decision.*



Results

- **Density Evolution versus our decoder performance**



Results

- *We expect*: **P < 1/2** ➔ recover all erased bits.

- *We got*: sharp degradation in the success rate around probability **P=0.43.**

Why ?

1. Our decoder is not ideal.
2. The code (H matrix) is not optimally.

Suggestions for continuing

- ✓ **Extending the check of each equation for more than one unknown bit.**

but….

- ❖ Complexity growing.
- ❖ The decoder would not fit a real time system requirements.

LDPC

Communication Project

The End.