Dynamic Spectrum Management Revisited

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Kplus
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Part I

Introduction

Spectrum Management

Definition

Spectrum management intends to minimize the effects of crosstalk noise and maximize the utilization of cable capacity

Method

Power Control and Power Allocation

Dynamic Spectrum Management (DSM)

- Optimized transmit PSD shapes designed based on actual network topology and crosstalk couplings as well as individual needs of each user
- Different *transmit* PSDs for each modem

DSM Levels

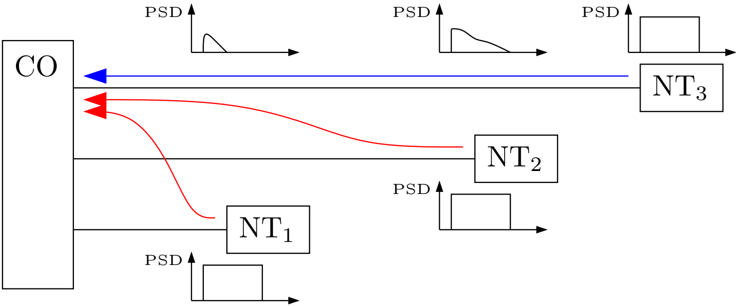
DSM Levels

- Level 0: No coordination (i.e static spectrum management)
- Level 1: Autonomous (single-user) power allocation aiming at crosstalk avoidance
- Level 2: Coordinated (multi-user) power allocation aiming at crosstalk avoidance
- Level 3: Multi-user transmission aiming at crosstalk mitigation (MIMO-DSL)

Centralized vs Decentralized

Identifying the Problem

"FEXT is not reciprocal" [BT contrib. to ETSI TM3 Dec. 1996]
... also known as the "Near-far problem"

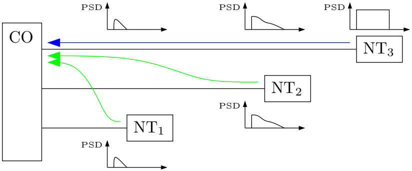


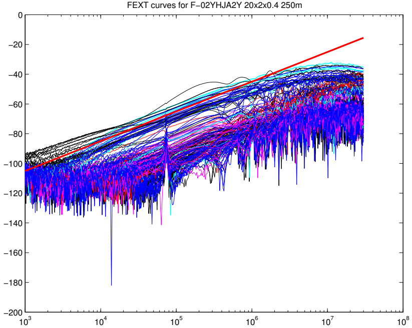
- Full-upstream transmission on short (near) loops results in high-level far-end crosstalk (FEXT) noise on long (far) loops.

Loop Interactions

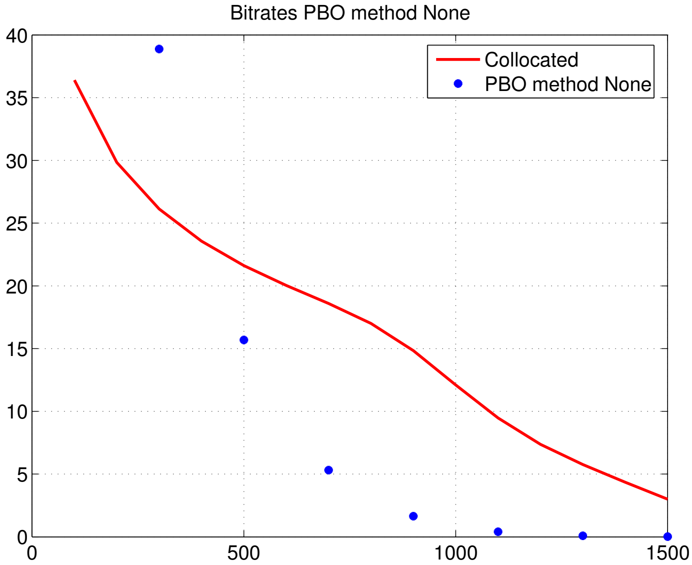
The level of FEXT noise will depend on

- Coupling strength between pairs depend on cable type, geometry, and gauge
- Coupling length (loop topology)
- Loop attenuation
- Frequency of interest
- PSD of the disturber



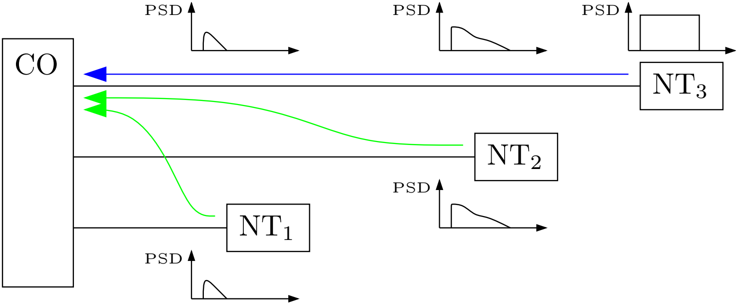


Bitrate Loss Appears...



Solution to the Near-Far Problem?

- Transmitters on shorter loops must reduce their PSD, that is, perform dynamic spectrum management
- For VDSL we call this reduction (upstream) power back-off (PBO)



Years of Discussion...

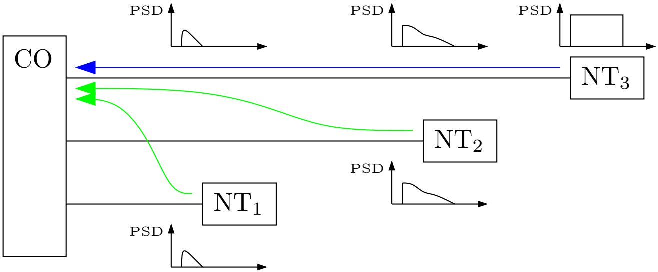
- What services to protect? – an operator issue
  - Equal pain (mixing services)
  - Long loop protection (maximum reach)
- What PSD shapes can be allowed? – a manufacturer problem
  - Is flat reduction enough?
  - Can it depend on loop length, measured noise, etc?
- What is optimal? – a research problem
  - Can we find the optimum without exhaustive search?

Common VDSL PBO Framework

Identify the target recieved PSD in the PBO methods and denote it by PSD_{REF}.

A common VDSL PBO framework

$$TxPSD(f, L_i) = \frac{PSD_{REF}(L_i)}{|H(f, L_i)|^2}$$



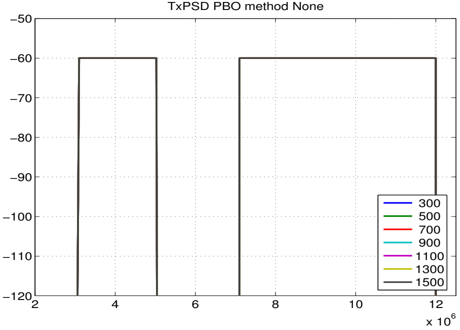
Simplest Function for PSD_{REF}?

Reference PSD (per upstream band)

$$PSD_{REF}(f) = \alpha + \beta\sqrt{f}$$

Transmit PSD (with a mask):

$$TxPSD(f, L_i) = min\left(\frac{PSD_{REF}(f)}{|H(f, L_i)|^2}, PSD_{STD}(f)\right)$$



PBO Parameter Selection

The goal is to optimize the PBO parameters:

$$\Phi = \{\alpha_{1U}, \beta_{1U}, \alpha_{2U}, \beta_{2U}{}^{1}\}$$

For a region (country) the optimal parameters depend on:

- Service requirements, $\mathcal{S}$, (protected bitrates)
- Network topology
- Alien noise environment
- Transmit PSD mask, $PSD_{STD}(f)$

Solved for a region in [Statovci et al., ICASSP 2006] using Nelder-Mead simplex search.

Power Control

Some of the algorithms proposed for the Power Control and Power Allocation problem in DSL:

- Iterative Water-Filling (IWF) - Distributed
  Yu and Ginis, 2002
- Optimal Spectrum Balancing (OSB) - Centralized
  Cendrillon et al., 2004
- Iterative Spectrum Balancing (ISB) - Centralized
  Cendrillon et al., 2005, Lui et al. 2005 (both at ICC2005)
- Mask-based Spectrum Balancing (MSB) - Centralized
  Statovci and Nordström, 2007
- Normalized-Rate Iterative Algorithm (NRIA) - Centralized
  Statovci and Nordström, 2004
- User-Unique Power Back-Off (UUPBO) - Centralized
  Statovci and Nordström, 2007

Calculation of Bitrates

Number of total bits per DMT symbol for a particular user u:

$$R_u = \sum_{n \in I} \log_2 \left(1 + \frac{\mathcal{H}_{uu}^n \mathcal{P}_u^n}{\Gamma \mathcal{N}_u^n} \right), \quad \text{with} \tag{1}$$

$$\mathcal{N}_u^n = \sum_{\substack{v=1 \\ v \neq u}}^{U} \mathcal{H}_{uv}^n \mathcal{P}_v^n + \mathcal{P}_{u,BGN}^n, \tag{2}$$

where,

- I denotes the set of subcarriers used in a particular transmission direction and it comprises N subcarrier
- Γ is the signal-to-noise ratio gap
- $\mathcal{N}_u^n$, $\mathcal{P}_u^n$, $\mathcal{P}_{u,BGN}^n$ denote the PSD of user u in subcarrier n of noise, transmit signal, and the sum of background and alien noises
- $\mathcal{H}_{uv}^n$ denotes the squared magnitude of channel transfer function from user v to user u, *i.e.*, it represents either the direct channel (with $v = u$), or far end crosstalk (FEXT) coupling

Assumptions

For the bitrate calculation in (1) to hold we have assumed:

- A frequency division duplex (FDD) transmission scheme (no NEXT)
- Certain bit-error rate (typ. 10^{-7}) and coding (in Γ)
- Synchronized modems (carrier independence)
- Perfect channel knowledge ($\mathcal{H}_{uv}^{n}$)

Optimization Criteria

$$\underset{\mathcal{P}_u^n;\forall u,n}{\text{maximize}} \sum_{u=1}^{U} w_u R_u, \tag{3a}$$

$$\text{subject to: } \sum_{n \in I} \mathcal{P}_u^n \leq T_u^{\max}, \quad \forall\, u, \tag{3b}$$

$$\mathcal{P}_u^n \geq 0, \forall\, u, \forall n \in I \tag{3c}$$

where,

- weights w_u represent priorities given to different users u. Without loss of generality, the weights can be selected such that $\sum_{u=1}^{U} w_u = 1$.
- $T_u^{\max}$ denotes the total power constraint for user u.

Difficult problem...

Non-linear non-convex optimization!

Dual Decomposition

Form the Lagrangian

$$L = \sum_{u=1}^{U} w_u R_u + \sum_{u=1}^{U} \lambda_u \left(T_u^{\max} - \sum_{n \in I} \mathcal{P}_u^n \right), \tag{4}$$

$$\underset{\mathcal{P}_u^n;\forall u,n}{\text{maximize}}\ L(w_u, \lambda_u, \mathcal{P}_u^n), \tag{5a}$$

$$\text{subject to: } \mathcal{P}_u^n \geq 0, \forall\, u, \forall n \in I \tag{5b}$$

$$\lambda_u \geq 0, \forall\, u. \tag{5c}$$

First? formulation [Lee 2002] , duality gap analysis [Yu&Cendrillon 2004]

Restating the Optimization Problem

By collecting the terms that belong to the same subcarrier equation (4) can be rewritten as

$$L = \sum_{n \in I} L^n + \sum_{u=1}^{U} \lambda_u T_u^{\max}, \tag{6}$$

where L^n is the Lagrangian on subcarrier n given by

$$L^n = \sum_{u=1}^{U} w_u R_u^n - \sum_{u=1}^{U} \lambda_u \mathcal{P}_u^n. \tag{7}$$

The optimization is now divided into N per-subcarrier optimization subproblems that are only related through the weighs w_u and Lagrangian multipliers λ_u.

Notes on this Optimization Problem Formulation

- The optimization has a complexity that scales linearly with the number of subcarriers.
- The problem still has an complexity that increases exponential with the number of users (lines).

Optimal Spectrum Balancing (OSB)

Optimal Spectrum Balancing solves this optimization problem

- by solving the decomposed (per tone) subproblems by an exhaustive search of optimal spectra.
- Optimal values of $\lambda_1, ..., \lambda_u$ can be found using bisection or subgradient search methods

[Cendrillon, 2004]

Optimal Spectrum Balancing Algorithm

Preset values: $R_u^{\text{target}}, T_u^{\max}, \ \forall u$

repeat

  for $n = 1$ to N **do**

    $P_1^n, \ldots, P_U^n = \text{maximize}_{\mathcal{P}_1^n, \cdots, \mathcal{P}_U^n} \sum_{u=1}^{U} (w_u R_u^n - \lambda_u \mathcal{P}_u^n)$

    {i.e., a U-dimensional exhaustive search }

    for $u = 1$ to U **do**

      adjust λ_u until the total power constraint is satisfied (or $\lambda_u = 0$)

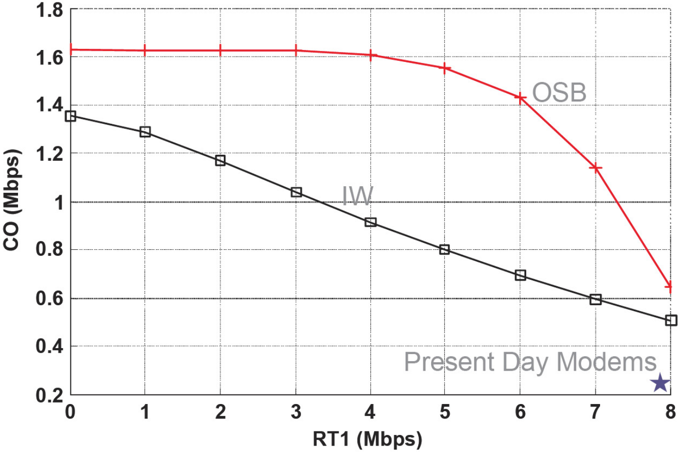
      adjust w_u until the target rate is satisfied

    end for

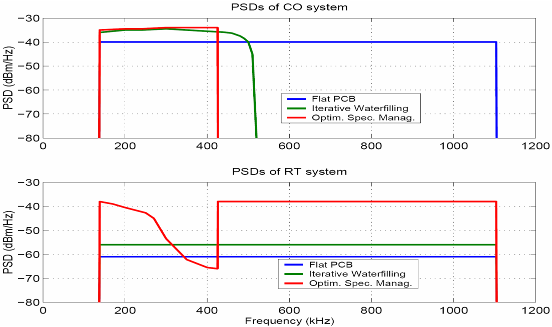
  end for

until convergence

OSB Performance



OSB Behavior



- CO will not transmit in high frequency due to bad direct channel
- RT experiences little crosstalk from CO.
- Iterative waterfilling can only adjust power through a constant λ^n
- OSB can find the optimal spectra on each tone.

Optimal Spectrum Balancing

- Pros
  - Solve a long-standing open problem
  - Find the global optimal solution (asymptotically)
  - Linear complexity in N
  - Nice as reference when less complex algorithms are developed
- Cons
  - Target rates needs to be known in advance
  - Exponential complexity in U
  - Numerical problems

Iterative Spectrum Balancing

- Users take turns to optimize their own PSD
- Each user $\tilde{u}$ solves the optimization problem, assuming fixed PSD of other users (cf. IWF)
  - Dual-decomposition leads again to one *nonconvex* subproblem per tone n
  - Find optimal $\mathcal{P}_{\tilde{u}}^{n}$ by a single dimension exhaustive search
  - Find optimal $\lambda_{\tilde{u}}$ by subgradient method

[Cendrillon ICC2005, Lui ICC2005]

Iterative Spectrum Balancing Algorithm

repeat
  for $u = 1$ to U **do**
    repeat
      Fix $P_j^n, \forall j \neq u$
      Fix $w_j, \forall j \neq u$
      $P_u^n = \arg\max_{\mathcal{P}_u^n} L^n$ for $n \in I$
      {i.e., a one dimensional exhaustive search }
      Update: $w_u = \left[w_u + \epsilon \left(R_u^{\text{target}} - \sum_{n \in I} R_u^n \right) \right]^+$
      Update: $\lambda_u = \left[\lambda_u + \epsilon \left(\sum_{n \in I} P_u^n - T_u^{\max} \right) \right]^+$
    until convergence for user u
  end for
until the PSDs of all users have reach a desired accuracy

$[]^+$: constraint to non-negative numbers

Iterative Spectrum Balancing

- Iterative solution to spectrum balancing problem
- Like IWF:
  - Optimize PSD of each user in turn
  - Low complexity ($O(NU^2)$ for ISB)
  - Tractable for large N
- Like OSB:
  - Uses weighted rate sum (avoids selfish-optimum of IWF)
  - Near-optimal performance
    100 - 150% gains over Iterative Waterfilling (for non-realistic case, but for typical cases it is more like 5 - 15%)
  - Target rates needs to be known in advance
  - Centralized solution

Open Issues OSB/ISB

- Convergence
  - Convergence not assured if PSD (or MAXBIT) constraint is assumed?
  - We see numerical instabilities (double precision not enough) using USB/ISB
- Ways to reduce complexity without sacrificing to much performance?

Mini Example

Looking at the i-th iteration, for a two-user case, the optimization problem for the first user on a particular subcarrier n is

$$\underset{\mathcal{P}_1^n}{\text{maximize}} \sum_{u=1}^{2} w_u R_u^n - \sum_{u=1}^{2} \lambda_u \mathcal{P}_u^n, \tag{8a}$$

$$\text{subject to: } \mathcal{P}_u^n \geq 0, \forall\, u, \forall n \in I \tag{8b}$$

This cost function is neither concave nor convex with respect to power allocation of a particular user [Cendrillon_ICC2005, Lui_ICC2005].

Mini Example - Some Iterations

iteration	u_1	u_2
$i-3$		all carriers used
$i-2$	total power not used to achieve R_1 target rate, thus $\lambda_1 = 0$	
$i-1$		one of the carriers not used† $P_2^n = 0$
i	Now, $\lambda_1 = 0$ and $P_2^n = 0$	

† This occurs whenever the SNR of the second user on the subcarrier n is low either due to high noise level or high channel attenuation

Mini Example - Divergence

A problem appears when $\mathcal{P}_2^n = 0$,† and $\lambda_1 = 0$!

The per tone Lagrangian in (8a) can be written out for our two-user case:

$$L^n = w_1 R_1^n + w_2 R_2^n - \lambda_1 \mathcal{P}_1^n - \lambda_2 \mathcal{P}_2^n \tag{9}$$

Thus, the optimization (8) becomes

$$\underset{\mathcal{P}_1^n}{\text{maximize}}\; w_1 \log_2 \left(1 + \frac{\mathcal{H}_{11}^n \mathcal{P}_1^n}{\Gamma \mathcal{P}_{1,BGN}^n} \right), \tag{10a}$$

$$\text{subject to: } \mathcal{P}_u^n \geq 0, \forall\, u, \forall n \in I \tag{10b}$$

Exhaustive search for $\mathcal{P}_1^n$ will take a while...

Mask-based Spectrum Balancing (MSB)

From the previous mini example we conclude that a PSD mask is always needed.

Why not completely base it on a mask constraint?

$$\underset{\mathcal{P}_u^n;\forall u,n}{\text{maximize}} \sum_{u=1}^{U} w_u R_u, \tag{11a}$$

$$\text{subject to: } 0 \leq \mathcal{P}_u^n \leq \mathcal{P}_u^{n,\max}, \forall\, u, \forall n \in I, \tag{11b}$$

We then have what we will call
Mask-based Spectrum Balancing (MSB)

[Statovci, Nordström, and Nilsson, "Spectrum Balancing for DSL with Restrictions on Maximum Transmit PSD", AccessNets 2007, Aug. 2007]

Mask-based Spectrum Balancing Algorithm

Preset values: $w_u, \ \forall u$

$P_u^{n,\max}, \ \forall u, \forall n \in I$ {mask constraints}

repeat

  for $u = 1$ to U **do**

    Calculate Noise $\mathcal{N}_u^n$ for $n \in I$ as in (2)

    $P_u^n = \arg\max_{\mathcal{P}_u^n} \sum_{u=1}^{U} w_u R_u^n$ for $n \in I$

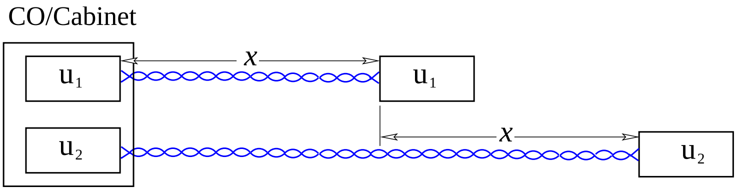
    {Solve by 1-D exhaustive search under PSD constraint constraint (11b)}

  end for

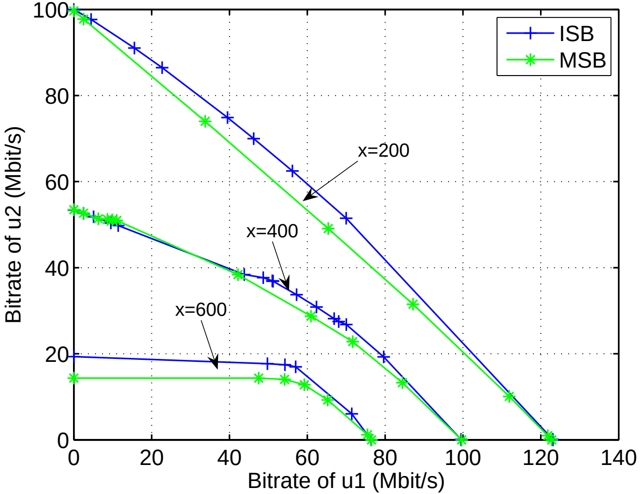
until the PSDs of all users have reach a desired accuracy

Comparison

Simulation scenario with two users:



Rate Region Comparison - MBS vs ISB

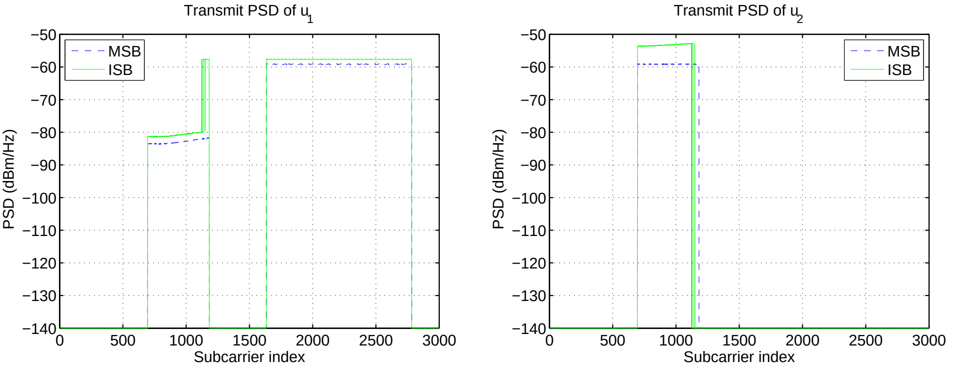


Rate Comparison - MBS vs ISB

Comparison of the MSB with the OSB for some particular pairs of bitrates.

Algorithm	Scenario (x in m)	User u_1 (Mbit/s)	User u_2 (Mbit/s)	Loss (%)
ISB	600	62.2	14.6	–
MSB	600	59.2	12.8	6.6
ISB	400	82.0	17.0	–
MSB	400	80.8	16.0	2.2
ISB	200	107.5	15.0	–
MSB	200	107.0	14.3	3.0

PSD Comparison - MBS vs ISB



Concluding MSB

We have found MSB:

- All current standards sets a PSD mask
- A PSD mask is needed for convergence reasons
- MSB needs significantly lower complexity compared to ISB, as indication: the simulation time to get a pair of bitrates is 3 seconds for MSB while ISB (for fixed weights) requires 114 seconds.
- It looses only a small amount, typically a few percent, of performance compared to ISB (due to less degrees of freedom)

Constraining PSD

Another way to speed up the optimization process is to constrain the PSD shapes.
For example, to the shapes used by the standardized (VDSL) power back-off.

Recall the received PSD:

$$\mathcal{P}_u^{\mathrm{R}}(f) = \alpha + \beta\sqrt{f}, \quad [\mathrm{dBm/Hz}] \tag{12}$$

The transmit PSD is then:

$$\mathcal{P}_u^n = \min\left\{\frac{\mathcal{P}_u^{n,\mathrm{R}}}{\mathcal{H}_{uu}^n}, \mathcal{P}_u^{n,\max}\right\}, \tag{13}$$

[Statovci, Nordström, Nilsson, ICASSP 2007]

Finding Target Rates

How to find good target rates?

Bitrate Relations

- Bit rates that can be supported for a particular network scenario are unknown in advance
- A concept of bitrate relations was first introduced with the normalized-rate iterative algorithm (NRIA) [Statovci and Nordström, ICC 2004]: $\frac{R_1}{p_1} = \frac{R_2}{p_2} = \ldots = \frac{R_U}{p_U}; \quad \sum_{u=1}^{U} p_u = 1$
- Example: Assume a cable with an upstream capacity of 60 Mbit/sec

User	User prior.	User bitrates	Norm. bitrates
u	p_u	R_u	R_u/p_u
1	1/3	20	60
2	2/3	40	60
Σ	1	60	

User-Unique Power Back-Off (UUPBO)

With the concept of PSD shaped as standard VDSL and with rate relations we have:

$$\underset{\Phi_1,\ldots,\Phi_U}{\text{maximize}} \sum_{u=1}^{U} R_u, \tag{14a}$$

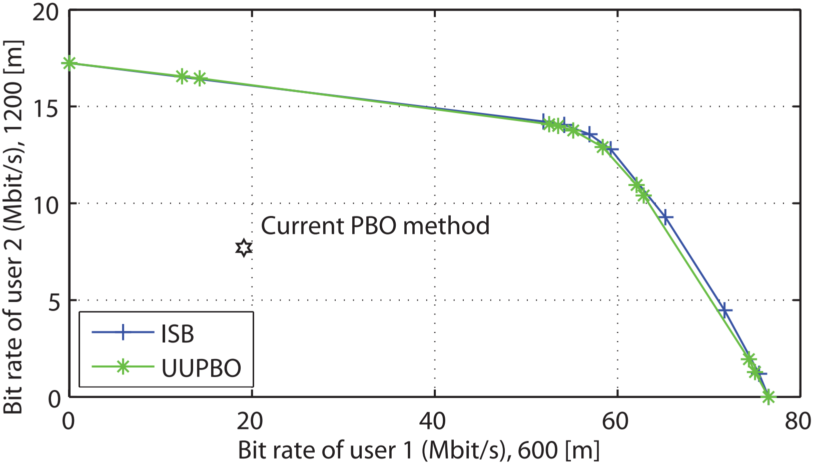
$$\text{subject to:} \frac{R_1}{p_1} = \frac{R_2}{p_2} = \ldots = \frac{R_U}{p_U}, \tag{14b}$$

$$\mathcal{P}_u^n = \min\left\{\frac{\mathcal{P}_u^{n,\mathrm{R}}}{\mathcal{H}_{uu}^n}, \mathcal{P}_u^{n,\max}\right\}, \quad \forall\, u, \forall n \in I \tag{14c}$$

$$\sum_{n \in I} \mathcal{P}_u^n \leq T_u^{\max}, \quad \forall\, u, \tag{14d}$$

where $T_u^{\max}$ denotes the maximum total power constraint for user u.

Comparing UUPBO with ISB



Notes on UUPBO

- Constraining PSD reduced complexity while still giving close to optimal performance
- Bit-rate relations helps the optimization process

Summary

We have now

- Looked at the (centralized) power allocation techniques, i.e., DSM level-2:
  - Optimal Spectrum Balancing (OSB)
  - Iterative Spectrum Balancing (ISB)
  - Mask-based Spectrum Balancing (MSB)
  - User-Unique Power Back-Off (UUPBO)
- Discussed some approaches to help the optimization process:
  - Importance of correct constraints (here: masks)
  - Usefulness of ”constraining” constraints (here: PSD shape)
  - Possibility to use different constraints (here: bitrate relations)

Our papers can be found via `http://xdsl.ftw.at/docs/papers/`