Interference Calculus

Part I:
Axiomatic Characterization of Interference in Wireless Networks

Part II:
Algorithms for Resource Allocation

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Outline

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   - Joint Beamforming and Power Allocation
   - SIR Balancing and Utility Optimization
2. Representation and Classification of Interference Functions
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3. SIR-Constrained Power Minimization
4. Utility Optimization Strategies
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Improving the Spectral Efficiency of Wireless Systems

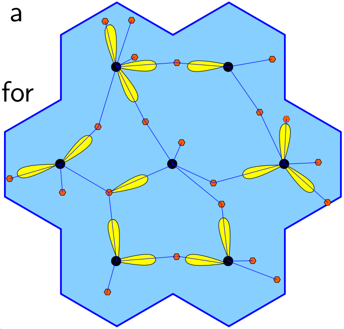
Motivation from information theory: the system performance is generally maximized by tolerating interference (in a controlled way)

⇒ the system can no longer be regarded as a collection of point-to-point links

⇒ techniques that were originally designed for wireline networks do not necessarily perform well in a wireless context

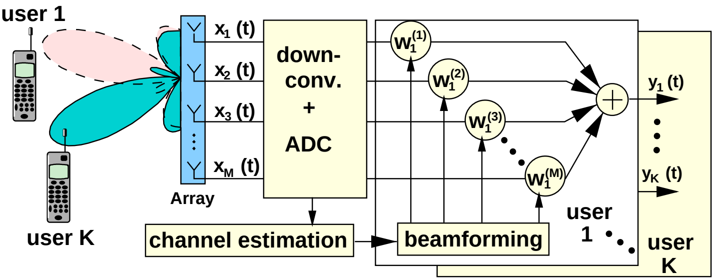
⇒ some interference-related issues:

- resource allocation
- interference mitigation
- adaptivity
- cooperation and interference coordination



Outline

Example: Interference Mitigation by Beamforming



- received array signal: $\mathbf{x} = \sum_{k=1}^{K} \mathbf{h}_k s_k + \mathbf{n}$
- output of the kth beamformer:

$$y_k = \mathbf{w}_k^H \mathbf{x} = \underbrace{\mathbf{w}_k^H \mathbf{h}_k s_k}_{\text{desired signal}} + \underbrace{\sum_{l \neq k} \mathbf{w}_k^H \mathbf{h}_l s_l}_{\text{interference}} + \underbrace{\mathbf{w}_k^H \mathbf{n}}_{\text{noise}}$$

Signal-to-Interference Ratio (SIR)

- Signal-to-Interference(-plus-Noise) Ratio:

$$\frac{\mathrm{E}\left[|\mathbf{w}_k^H \mathbf{h}_k s_k|^2\right]}{\mathrm{E}\left[|\mathbf{w}_k^H(\mathbf{x} - \mathbf{h}_k s_k)|^2\right]} \quad \begin{matrix} \leftarrow \text{useful power} \\ \leftarrow \text{interference+noise power} \end{matrix}$$

$$= \frac{p_k |\mathbf{w}_k^H \mathbf{h}_k|^2}{\mathbf{w}_k^H \left(\sigma_n^2 \mathbf{I} + \sum_{l \neq k} p_l \mathbf{h}_l \mathbf{h}_l^H\right) \mathbf{w}_k}$$

where $p_k = \mathrm{E}[|s_k|^2]$ is the transmit power of user k

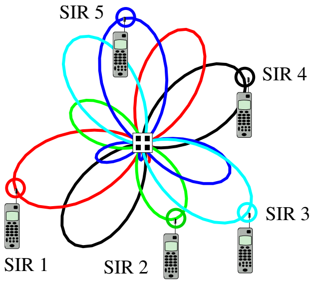
- this is maximized by $\mathbf{w}_k^* = \left(\sigma_n^2 \mathbf{I} + \sum_{l \neq k} p_l \mathbf{h}_l \mathbf{h}_l^H\right)^{-1} \cdot \mathbf{h}_k$

$$\Rightarrow \quad \mathrm{SIR}_k^{\max}(\mathbf{p}) = p_k \mathbf{h}_k^H \left(\sigma_n^2 \mathbf{I} + \sum_{l \neq k} p_l \mathbf{h}_l \mathbf{h}_l^H\right)^{-1} \mathbf{h}_k$$

Max-SINR Beamforming ("Optimum Combining")

signal-to-interference-(plus-noise) ratios (SIR) depend on the Tx power allocation

$$\mathbf{p} = [\underbrace{p_1, p_2, \ldots, p_K}_{Tx\ powers}, \underbrace{\sigma_n^2}_{noise}]^T$$



➠ residual interference:

$$\mathcal{I}_k(\mathbf{p}) = \frac{1}{\mathbf{h}_k^H \left(\sigma_n^2 \mathbf{I} + \sum_{l \neq k} p_l \mathbf{h}_l \mathbf{h}_l^H\right)^{-1} \mathbf{h}_k}$$

- the function $\mathcal{I}_k(\mathbf{p})$ has a "nice" analytical structure (concave, non-negative,monotonic,...), which has facilitated many interesting results and algorithms in the past
- is there a more general underlying concept?

Spatial Matched Filter

- the matched filter $\mathbf{w}_k = \mathbf{h}_k / \|\mathbf{h}_k\|_2$ is a single user receiver
- assuming quasi-static channels, we have a constant link gain matrix

$$\mathbf{V} = [\mathbf{v}_1, \ldots, \mathbf{v}_K]^T \geq 0$$

➠ the interference of the kth user is

$$\mathcal{I}_k(\mathbf{p}) = \mathbf{p}^T \mathbf{v}_k$$

- this linear interference function has a longstanding tradition in power control theory [Aein'73]

General Interference Functions

Definition

We say that $\mathcal{I} : \mathbb{R}_+^K \mapsto \mathbb{R}_+$ is an interference function if it fulfills the axioms:

A1	(non-negativeness)	$\mathcal{I}(\mathbf{p}) \geq 0$	
A2	(scale invariance)	$\mathcal{I}(\alpha\mathbf{p}) = \alpha\mathcal{I}(\mathbf{p})$	$\forall\alpha \in \mathbb{R}_+$
A3	(monotonicity)	$\mathcal{I}(\mathbf{p}) \geq \mathcal{I}(\mathbf{p}')$	if $\mathbf{p} \geq \mathbf{p}'$

- this framework generalizes the framework of standard interference functions [Yates'95]
- the beamforming example is a special case of this framework

Outline

The Downlink Power Minimization Problem

- joint optimization of beamformers $\mathbf{u}_1, \ldots, \mathbf{u}_K$ and powers $\mathbf{p} = [p_1, \ldots, p_K]$, for given channels $\mathbf{h}_1, \ldots, \mathbf{h}_K$
- problem formulation: achieve SIR targets $\boldsymbol{\gamma} = [\gamma_1, \ldots, \gamma_K]$ with minimum total power:

$$\min_{\mathbf{p}>0, \mathbf{u}_1, \ldots, \mathbf{u}_K} \quad \sum_{i=1}^{K} p_i$$

$$\text{s.t.} \qquad \frac{p_k |\mathbf{u}_k^H \mathbf{h}_k|^2}{\sum_{l \neq k} p_l |\mathbf{u}_l^H \mathbf{h}_k|^2 + \sigma_n^2} \geq \gamma_k, \quad \forall k = 1, \ldots, K$$

$$\|\mathbf{u}_k\| = 1$$

Different Approaches to Downlink Beamforming

- [Rashid-Farrokhi/Tassiulas/Liu,1998]
  - fixed-point iteration
  - general approach
- [Bengtsson/Ottersten,1999]
  - semidefinite programming, interior-point algorithms
  - exploits the special structure of the beamforming problem

  convexity is commonly considered as the dividing line between "easy" and "difficult" problems.
- special properties of interference functions (axioms A1-A3) also enable "easy" solutions

Reformulation Based on Interference Functions

- exploiting uplink/downlink duality, the problem can be rewritten in terms of uplink interference functions

$$\min_{\mathbf{p}>0} \sum_{l=1}^{K} p_l \quad \text{s.t. } p_k \geq \gamma_k \mathcal{I}_k(\mathbf{p}),\ k = 1, 2, \ldots, K \ ,$$

$$\text{where} \quad \mathcal{I}_k(\mathbf{p}) = \frac{1}{\mathbf{h}_k^H \left(\sigma_n^2 \mathbf{I} + \sum_{l \neq k} p_l \mathbf{h}_l \mathbf{h}_l^H\right)^{-1} \mathbf{h}_k} \ .$$

the special structure of $\mathcal{I}_k(\mathbf{p})$ can be exploited
(a globally convergent algorithm will be discussed in part II)

Outline

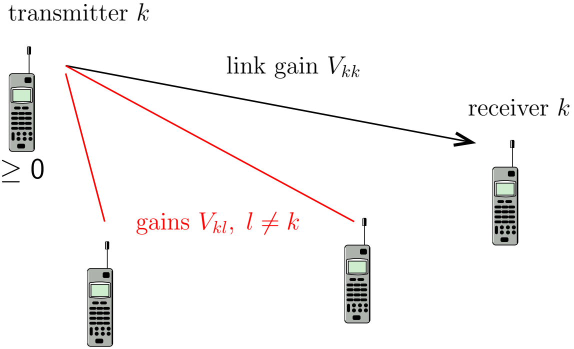
The Linear Interference Model

- transmit power vector $\mathbf{p} = [p_1, \ldots, p_K]^T$ (power allocation)
- non-negative irreducible link gain matrix

  $$\mathbf{V} = [\mathbf{v}_1, \ldots, \mathbf{v}_K]^T \geq 0$$

- interference of the kth user:

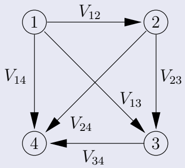
  $$\mathcal{I}_k(\mathbf{p}) = \mathbf{p}^T \mathbf{v}_k$$



Irreducibility

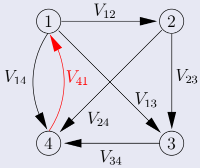
V is reducible ⇔ the directed graph is not fully connected

$$\mathbf{V} = \begin{bmatrix} 0 & V_{12} & V_{13} & V_{14} \\ 0 & 0 & V_{23} & V_{24} \\ 0 & 0 & 0 & V_{34} \\ 0 & 0 & 0 & 0 \end{bmatrix}$$



V is irreducible ⇔ the directed graph is fully connected

$$\mathbf{V} = \begin{bmatrix} 0 & V_{12} & V_{13} & V_{14} \\ 0 & 0 & V_{23} & V_{24} \\ 0 & 0 & 0 & V_{34} \\ V_{41} & 0 & 0 & 0 \end{bmatrix}$$



SIR Balancing and Perron-Frobenius Theory

- SIR feasible set:

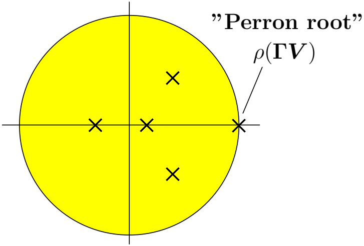
$$\mathcal{S} = \{\gamma : \rho(\mathbf{\Gamma V}) \leq 1\}$$

where $\mathbf{\Gamma} = \mathrm{diag}\{\gamma\}$ and

$$\rho(\mathbf{\Gamma V}) = \inf_{\mathbf{p}>0} \max_{k} \frac{[\mathbf{\Gamma V p}]_k}{p_k}$$

(Collatz/Wielandt)

illustrating example: eigenvalues of the non-neg. matrix ΓV in the complex plane



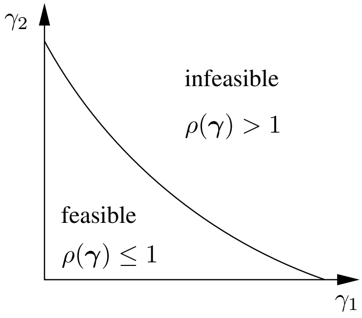
- if **V** is irreducible , then the SIR balancing problem is solved by the unique principal eigenvector associated with the spectral radius

$$\mathbf{\Gamma V p}^* = \rho(\mathbf{\Gamma V})\mathbf{p}^*$$

The SIR Feasible Region of the Linear Model

- observation: the function $\rho(\boldsymbol{\gamma}) := \rho(\boldsymbol{\Gamma}\mathbf{V}) = \rho(\mathrm{diag}\{\boldsymbol{\gamma}\}\mathbf{V})$ is an interference function
- SIR requirements $\boldsymbol{\gamma}$ are jointly feasible if $\rho(\boldsymbol{\gamma}) \leq 1$
- the SIR region is defined as

$$\mathcal{S} = \{\boldsymbol{\gamma} \in \mathbb{R}^K_+ : \rho(\boldsymbol{\gamma}) \leq 1\}$$



➠ later, it will be shown that every SIR region is a sub-level set of an interference function

➠ interference calculus is not restricted to power control problems, another application is the analysis of "performance tradeoff regions"

SIR Balancing with Adaptive Beamforming

- [Gerlach/Paulraj'96] have studied the problem of maximizing the minimum SIR (also referred to as SIR balancing)

$$\max_{\mathbf{p}>0,\mathbf{u}_1,\dots,\mathbf{u}_K} \left(\min_{1\le k\le K} \frac{p_k}{\sum_{l\neq k} p_l |\mathbf{u}_l^H \mathbf{h}_k|^2} \right) \quad \text{s.t.} \;\; |\mathbf{u}_k^H \mathbf{h}_k|^2 = 1$$

- [Montalbano/Slock'98]: uplink/downlink duality leads to the problem of Perron root optimization

$$\rho_{opt}(\gamma) = \min_{\mathbf{u}=\{\mathbf{u}_1,\dots,\mathbf{u}_K\}} \quad \rho\left(\begin{bmatrix} \gamma_1 & & 0 \\ & \ddots & \\ 0 & & \gamma_K \end{bmatrix} \cdot \mathbf{V}(\mathbf{u}) \right)$$

where $\rho(\cdot)$ is the spectral radius ("Perron root") and $\mathbf{V}(\mathbf{u})$ is a beamformer-dependent coupling matrix

SIR Balancing – Reformulation

- the Perron root minimization problem can be rewritten as

$$\rho_{opt}(\gamma) = \inf_{\mathbf{p}>0} \max_{1\leq k\leq K} \frac{\gamma_k \mathcal{I}_k(\mathbf{p})}{p_k}$$

$$\text{where} \quad \mathcal{I}_k(\mathbf{p}) = \min_{\mathbf{u}_k} \mathbf{p}^T \mathbf{v}_k(\mathbf{u}_k)$$

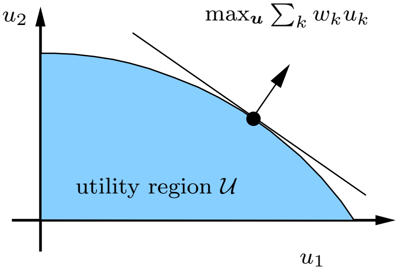
- the optimum $\rho_{opt}(\gamma)$ is a single criterion for the joint quality of all K users
- a globally convergent algorithm can be derived by exploiting that $\rho_{opt}(\gamma)$ is a concave interference function (in part II)

Utility Optimization

- maximize the weighted sum utility:

$$\mathcal{I}_{\mathcal{U}}(\mathbf{w}) = \max_{\mathbf{u}\in\mathcal{U}} \sum_{k=1}^{K} w_k u_k$$

- $\mathcal{I}_{\mathcal{U}}(\mathbf{w})$ is a convex interference function



Discussion

The examples show that there exist many different types of interference functions, which are useful for different applications, e.g.,

- physical layer modeling
- resource allocation
- fairness

Is there a unifying framework for interference functions?

Next, we discuss the structure of different classes of interference functions and applications

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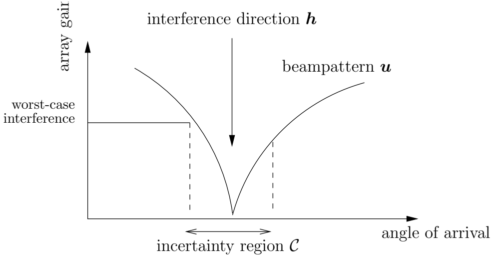
- this framework generalizes the framework of standard interference functions [Yates'95]
- the beamforming example is a special case of this framework

Example: Robust Nullsteering

interference can be reduced by nullsteering beamforming:

- assume that the interference direction is only known up to an uncertainty c from a region $\mathcal{C}$



- the beamformer $\mathbf{u}$ minimizes the worst-case interference power:

$$\mathcal{I}(\mathbf{p}) = \min_{\|\mathbf{u}\|=1} \left(\max_{c \in \mathcal{C}} \sum_l p_l |\mathbf{u}^H \mathbf{h}_l(c)|^2 \right)$$

$\Rightarrow$ this is also an interference function (A1–A3 fulfilled)

The Impact of Noise

- in order to include noise, we consider the extended power allocation $\mathbf{p} = \begin{bmatrix} \tilde{\mathbf{p}} \\ p_{K+1} \end{bmatrix}$ where the last component p_{K+1} stands for noise power
- an additional strict monotonicity property is required:

$$p_{K+1} > p'_{K+1}, \quad \text{with } \mathbf{p} \geq \mathbf{p}' \quad \Rightarrow \quad \mathcal{I}(\mathbf{p}) > \mathcal{I}(\mathbf{p}')$$

- If $p_{K+1} > 0$ is constant, and , then $\mathcal{I}(\tilde{\mathbf{p}})$ is 'standard' as defined in [Yates'95]

Fixed-Point Iteration

For standard interference functions it was shown [Yates'95]

If target SIR $\boldsymbol{\gamma} = [\gamma_1, \ldots, \gamma_K]$ are feasible, i.e., $C(\boldsymbol{\gamma}) \leq 1$, under a sum-power constraint, then for an arbitrary initialization $\mathbf{p}^{(0)} \geq 0$, the iteration

$$p_k^{(n+1)} = \gamma_k \cdot \mathcal{I}_k(\mathbf{p}^{(n)}), \quad k = 1, 2, \ldots, K$$

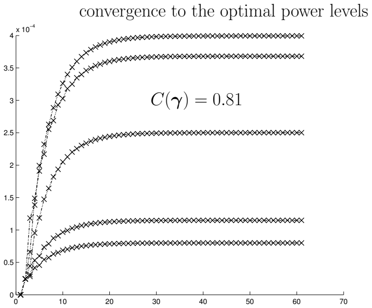
converges to the optimum of the power minimization problem

$$\inf_{\mathbf{p}>0} \sum_{k=1}^{K} p_k \quad \text{s.t. } \frac{p_k}{\mathcal{I}_k(\mathbf{p})} \geq \gamma_k, \ \forall k \ ,$$

Properties of the Fixed-Point Iteration

The fixed-point iteration has the following properties:

- component-wise monotonicity
- optimum achieved iff $p_k^{(n+1)} = \gamma_k \mathcal{I}_k(\mathbf{p}^{(n)}), \forall k$
- optimizer $\lim_{n\to\infty} \mathbf{p}^{(n)}$ is unique



The Weighted Max-Min Optimum $C(\gamma)$

- example: K interference functions $\mathcal{I}_1, \ldots, \mathcal{I}_K$ and weighting factors $\boldsymbol{\gamma} = [\gamma_1, \ldots, \gamma_K]$ (e.g. SIR requirements). The optimum of the weighted SIR balancing problem is

$$C(\gamma) = \inf_{\mathbf{p}>0} \left(\max_{1 \leq k \leq K} \frac{\gamma_k \mathcal{I}_k(\mathbf{p})}{p_k} \right)$$

- SIR feasible region

$$\mathcal{S} = \{\gamma : C(\gamma) \leq 1\}$$

➠ level set of the interference function $C(\gamma)$

Comparison of Min-Max and Max-Min Balancing

- alternative approach to SIR balancing:

$$c(\gamma) = \sup_{\mathbf{p}>0}\Big(\min_{1\le k\le K} \frac{\gamma_k \mathcal{I}_k(\mathbf{p})}{p_k}\Big) = \sup_{\mathbf{p}>0}\Big(\min_{1\le k\le K} \frac{\gamma_k}{\tilde{\gamma}_k(\mathbf{p})}\Big)$$

  in general, $c(\gamma) \le C(\gamma)$. ➠ fairness gap

- consider a fixed coupling matrix $\mathbf{V} \in \mathbb{R}_+^{K\times K}$. Under special conditions ($\mathbf{V}$ irreducible), both max-min and min-max fairness equal the spectral radius ρ:

$$C(\gamma) = \rho(\mathrm{diag}\{\gamma\}\mathbf{V}) = \inf_{\mathbf{p}>0} \max_{k} \frac{\gamma_k[\mathbf{V}\mathbf{p}]_k}{p_k} = c(\gamma)$$

Discussion

- $C(\gamma)$ and $c(\gamma)$ are constructed by underlying interference functions $\mathcal{I}_1, \ldots, \mathcal{I}_K$
  ➠ certain important operations are closed within the framework of interference functions
- for all general interference functions (only the basic properties A1–A3 fulfilled) algorithmic solutions exist (e.g. fixed point iteration)
- but more efficient solutions can be designed by exploiting the structure of the interference functions

Representation of General Interference Functions

Theorem

Let $\mathcal{I}$ be an arbitrary interference function, then

$$\mathcal{I}(\mathbf{p}) = \min_{\hat{\mathbf{p}} \in \underline{L}(\mathcal{I})} \max_k \frac{p_k}{\hat{p}_k} = \max_{\hat{\mathbf{p}} \in \overline{L}(\mathcal{I})} \min_k \frac{p_k}{\hat{p}_k}$$

- $\mathcal{I}(\mathbf{p})$ can always be represented as the optimum of a weighted max-min (or min-max) optimization problem
- The weights $\hat{\mathbf{p}}$ are elements of convex/concave level sets

$$\underline{L}(\mathcal{I}) = \{\hat{\mathbf{p}} > 0 : \mathcal{I}(\hat{\mathbf{p}}) \le 1\}$$
$$\overline{L}(\mathcal{I}) = \{\hat{\mathbf{p}} > 0 : \mathcal{I}(\hat{\mathbf{p}}) \ge 1\}$$

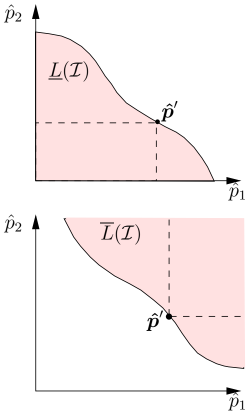
Interference Functions and Utility/Cost Regions

- the set $\underline{L}(\mathcal{I})$ is closed bounded and monotonic decreasing

$$\hat{\mathbf{p}} \leq \hat{\mathbf{p}}' , \quad \hat{\mathbf{p}}' \in \underline{L}(\mathcal{I}) \quad \Longrightarrow \quad \hat{\mathbf{p}} \in \underline{L}(\mathcal{I})$$

- the set $\overline{L}(\mathcal{I})$ is closed and monotonic increasing

$$\hat{\mathbf{p}} \geq \hat{\mathbf{p}}' , \quad \hat{\mathbf{p}}' \in \overline{L}(\mathcal{I}) \quad \Longrightarrow \quad \hat{\mathbf{p}} \in \overline{L}(\mathcal{I})$$



➠ every interference function can be interpreted as a utility/cost resource allocation problem

Outline

Concave Interference Functions

Definition

We say that $\mathcal{I} : \mathbb{R}_+^K \mapsto \mathbb{R}_+$ is a concave interference function if it fulfills the axioms:

A1	(non-negativeness)	$\mathcal{I}(\mathbf{p}) \geq 0$	
A2	(scale invariance)	$\mathcal{I}(\alpha\mathbf{p}) = \alpha\mathcal{I}(\mathbf{p})$	$\forall\alpha \in \mathbb{R}_+$
A3	(monotonicity)	$\mathcal{I}(\mathbf{p}) \geq \mathcal{I}(\mathbf{p}')$	if $\mathbf{p} \geq \mathbf{p}'$
C1	(concavity)	$\mathcal{I}(\mathbf{p})$ is concave on $\mathbb{R}_+^K$	

Examples for Concave Interference Functions

- beamforming:

$$\mathcal{I}_k(\mathbf{p}) = \frac{1}{\mathbf{h}_k^H \left(\sigma_n^2 \mathbf{I} + \sum_{l \neq k} p_l \mathbf{h}_l \mathbf{h}_l^H\right)^{-1} \mathbf{h}_k}$$

- generalization: receive strategy z_k

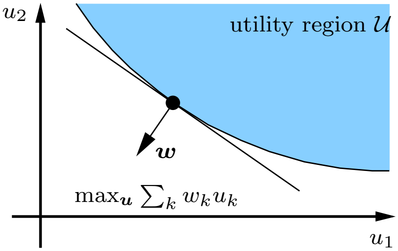
$$\mathcal{I}_k(\mathbf{p}, \sigma_n^2) = \min_{z_k \in \mathcal{Z}_k} \Big(\underbrace{\mathbf{p}^T \mathbf{v}(z_k)}_{\textit{Interference}} + \underbrace{\sigma_n^2 n_k(z_k)}_{\textit{Noise}} \Big) , \quad k = 1, 2, \ldots, K$$

Cost/Loss Minimization

- minimize the weighted sum cost:

$$\mathcal{I}_{\mathcal{U}}(\mathbf{w}) = \max_{\mathbf{u}\in\mathcal{U}} \sum_{k=1}^{K} w_k u_k$$

- $\mathcal{I}_{\mathcal{U}}(\mathbf{w})$ is a concave interference function



Representation of Concave Interference Functions

Theorem

Let $\mathcal{I}(\mathbf{p})$ be an arbitrary concave interference function, then

$$\mathcal{I}(\mathbf{p}) = \min_{\mathbf{w} \in \mathcal{N}_0(\mathcal{I})} \sum_{k=1}^{K} w_k p_k \ , \quad \textit{for all } \mathbf{p} > 0.$$

where

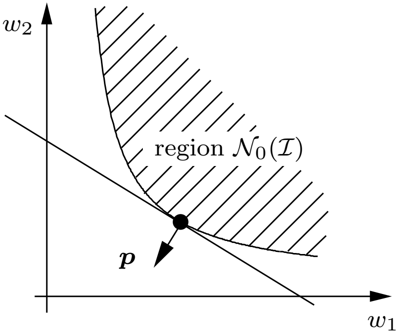
$$\mathcal{N}_0(\mathcal{I}) = \{\mathbf{w} \in \mathbb{R}_+^K : \underline{\mathcal{I}}^*(\mathbf{w}) = 0\}$$

and $\underline{\mathcal{I}}^*(\mathbf{w}) = \inf_{\mathbf{p}>0} \left(\sum_{l=1}^{K} w_l p_l - \mathcal{I}(\mathbf{p}) \right)$ *is the conjugate of* $\mathcal{I}$.

Interpretation of Concave Interference Functions

$$\mathcal{I}(\mathbf{p}) = \min_{\mathbf{w} \in \mathcal{N}_0(\mathcal{I})} \sum_{k=1}^{K} w_k p_k$$

- the set $\mathcal{N}_0(\mathcal{I})$ is closed, convex, and monotonic increasing, i.e., $\mathbf{w} \in \mathcal{N}_0(\mathcal{I})$ implies $\mathbf{w}' \geq \mathbf{w}$ belongs to $\mathcal{N}_0(\mathcal{I})$
- any concave interference function can be interpreted as the solution of a loss/cost minimization problem



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Convex Interference Functions

Definition

We say that $\mathcal{I} : \mathbb{R}_+^K \mapsto \mathbb{R}_+$ is a convex interference function if it fulfills the axioms:

A1	(non-negativeness)	$\mathcal{I}(\mathbf{p}) \geq 0$
A2	(scale invariance)	$\mathcal{I}(\alpha\mathbf{p}) = \alpha\mathcal{I}(\mathbf{p}) \quad \forall\alpha \in \mathbb{R}_+$
A3	(monotonicity)	$\mathcal{I}(\mathbf{p}) \geq \mathcal{I}(\mathbf{p}') \quad$ if $\mathbf{p} \geq \mathbf{p}'$
C2	(convexity)	$\mathcal{I}(\mathbf{p})$ is convex on $\mathbb{R}_+^K$

Example: Robustness

- Another example is the worst-case model

$$\mathcal{I}_k(\mathbf{p}) = \max_{c_k \in \mathcal{C}_k} \mathbf{p}^T \mathbf{v}(c_k), \quad \forall k \ ,$$

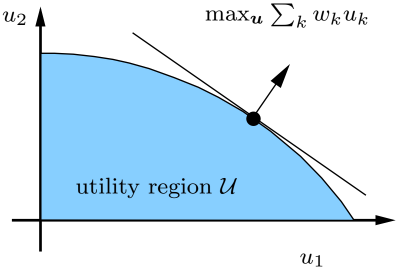
  where the parameter c_k models an 'uncertainty' (e.g. caused by channel estimation errors or system imperfections).
- the optimization is over a compact uncertainty region $\mathcal{C}_k$
- $\mathcal{I}_k(\mathbf{p})$ is a convex interference function

Utility Maximization

- maximize the weighted sum utility:

$$\mathcal{I}_{\mathcal{U}}(\mathbf{w}) = \max_{\mathbf{u}\in\mathcal{U}} \sum_{k=1}^{K} w_k u_k$$

- $\mathcal{I}_{\mathcal{U}}(\mathbf{w})$ is a convex interference function



Representation of Convex Interference Functions

Theorem

Let $\mathcal{I}(\mathbf{p})$ be an arbitrary convex interference function, then

$$\mathcal{I}(\mathbf{p}) = \max_{\mathbf{w} \in \mathcal{W}_0(\mathcal{I})} \sum_{k=1}^{K} w_k \cdot p_k \,, \quad \textit{for all } \mathbf{p} > 0.$$

where

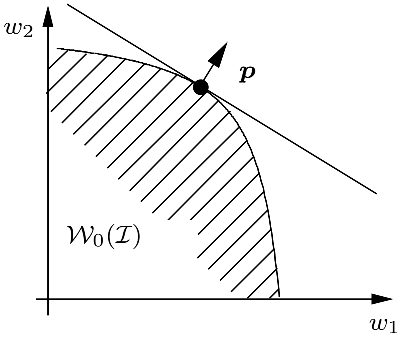
$$\mathcal{W}_0(\mathcal{I}) = \{\mathbf{w} \in \mathbb{R}_+^K : \bar{\mathcal{I}}^*(\mathbf{w}) = 0\}$$

and $\bar{\mathcal{I}}^*(\mathbf{w}) = \sup_{\mathbf{p}>0}\left(\sum_{l=1}^{K} w_l p_l - \mathcal{I}(\mathbf{p})\right)$ *is the conjugate of* $\mathcal{I}$.

Interpretation of Convex Interference Functions

$$\mathcal{I}(\mathbf{p}) = \max_{\mathbf{w} \in \mathcal{W}_0(\mathcal{I})} \sum_{k=1}^{K} w_k \cdot p_k$$

- the set $\mathcal{W}_0(\mathcal{I})$ is closed, convex, and monotonic decreasing, i.e., $\mathbf{w} \in \mathcal{W}_0(\mathcal{I})$ implies $\mathbf{w}' \leq \mathbf{w}$ belongs to $\mathcal{W}_0(\mathcal{I})$
- any convex interference function can be interpreted as the solution of a utility maximization problem



Outline

Log-Convex Interference Functions

Definition

We say that $\mathcal{I} : \mathbb{R}_+^K \mapsto \mathbb{R}_+$ is a log-convex interference function if it fulfills the axioms:

A1	(non-negativeness)	$\mathcal{I}(\mathbf{p}) \geq 0$	
A2	(scale invariance)	$\mathcal{I}(\alpha\mathbf{p}) = \alpha\mathcal{I}(\mathbf{p})$	$\forall\alpha \in \mathbb{R}_+$
A3	(monotonicity)	$\mathcal{I}(\mathbf{p}) \geq \mathcal{I}(\mathbf{p}')$	if $\mathbf{p} \geq \mathbf{p}'$
C3	(log-convexity)	$\mathcal{I}_k(\mathrm{e}^{\mathbf{s}})$ is log-convex on $\mathbb{R}^K$	

Log-Convexity

Let $f(\mathbf{s}) := \mathcal{I}(\exp\{\mathbf{s}\})$. The function $f : \mathbb{R}^K \mapsto \mathbb{R}_+$ is said to be log-convex on $\mathbb{R}^K$ if $\log f$ is convex, i.e.,

$$\log f\big((1-\lambda)\hat{\mathbf{s}}+\lambda\check{\mathbf{s}}\big) \leq (1-\lambda)\log f(\hat{\mathbf{s}})+\lambda\log f(\check{\mathbf{s}}), \quad \forall\lambda \in (0,1),\ \hat{\mathbf{s}},\check{\mathbf{s}} \in \mathbb{R}^K$$

taking exp on both sides, this is equivalent to [e.g. Boyd/Vandenbergh]

$$f\big((1-\lambda)\hat{\mathbf{s}} + \lambda\check{\mathbf{s}}\big) \leq f(\hat{\mathbf{s}})^{1-\lambda} f(\check{\mathbf{s}})^{\lambda}$$

Example

- Let $\mathcal{I}_1, \ldots, \mathcal{I}_K$ be log-convex interference functions, then the SIR-balancing optimum

$$C(\gamma) = \inf_{\mathbf{p}>0} \Big(\max_{1\leq k\leq K} \frac{\gamma_k \mathcal{I}_k(\mathbf{p})}{p_k} \Big)$$

is a log-convex interference function

Basic Properties

- the properties of log-convex interference functions are preserved under certain operations
- example: let $\mathcal{I}_1$ and $\mathcal{I}_2$ be log-convex interference functions, and

$$\mathcal{I}'(\mathbf{p}) = \alpha_1 \mathcal{I}_1(\mathbf{p}) + \alpha_2 \mathcal{I}_2(\mathbf{p}), \quad \alpha_1, \alpha_2 \in \mathbb{R}_+$$
$$\mathcal{I}''(\mathbf{p}) = \big(\mathcal{I}_1(\mathbf{p})\big)^{\alpha} \cdot \big(\mathcal{I}_1(\mathbf{p})\big)^{1-\alpha}, \quad \alpha \in [0,1]$$

➠ $\mathcal{I}'(\mathbf{p})$ and $\mathcal{I}''(\mathbf{p})$ are log-convex interference functions (note: this is not valid for log-concave interference functions)

➠ log-convex interference functions have a rich analytical and algebraic structure.

Representation of Log-Convex Interference Functions

Theorem

Every log-convex interference function $\mathcal{I}(\mathbf{p})$*, with* $\mathbf{p} > 0$*, can be represented as*

$$\mathcal{I}(\mathbf{p}) = \max_{\mathbf{w} \in \mathcal{L}(\mathcal{I})} \Big(f_{\mathcal{I}}(\mathbf{w}) \cdot \prod_{l=1}^{K} (p_l)^{w_l} \Big) .$$

where $$f_{\mathcal{I}}(\mathbf{w}) = \inf_{\mathbf{p}>0} \frac{\mathcal{I}(\mathbf{p})}{\prod_{l=1}^{K} (p_l)^{w_l}} , \quad \mathbf{w} \in \mathbb{R}_{+}^{K}, \quad \sum_k w_k = 1$$

$$\mathcal{L}(\mathcal{I}) = \big\{ \mathbf{w} \in \mathbb{R}_{+}^{K} : f_{\mathcal{I}}(\mathbf{w}) > 0 \big\}$$

Connection between Convex and Log-Convex Functions

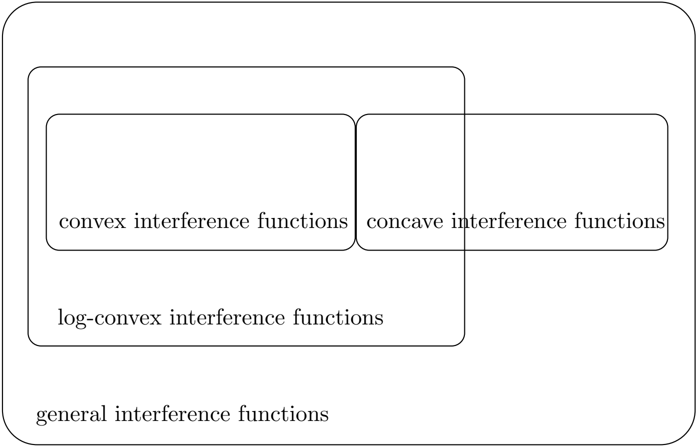
- every convex function $\mathcal{I}(\mathbf{p})$ can be expressed as

$$\mathcal{I}(\mathbf{p}) = \max_{\mathbf{w} \in \mathcal{W}_0} \sum_k w_k p_k$$

$\log \sum_k w_k \mathrm{e}^{s_k}$ is convex
$\Longrightarrow$ $\log \max_{\mathbf{w} \in \mathcal{W}_0} \sum_k w_k \mathrm{e}^{s_k}$ is convex
$\Longrightarrow$ $\mathcal{I}(\mathrm{e}^{\mathbf{s}})$ is log-convex

- if $\mathcal{I}(\mathbf{p})$ is convex then $\mathcal{I}(\mathrm{e}^{\mathbf{s}})$ is log-convex
  (but the converse is not true)

Categories of Interference Functions



Outline

Fixed-Point Iteration

- consider the power minimization problem

$$\inf_{\mathbf{p}>0} \sum_{k=1}^{K} p_k \quad \text{s.t. } \frac{p_k}{\mathcal{I}_k(\mathbf{p})} \geq \gamma_k,\ \forall k\ ,$$

with feasible) target SIR $\boldsymbol{\gamma} = [\gamma_1, \ldots, \gamma_K]$ and interference functions $\mathcal{I}_1, \ldots, \mathcal{I}_K$.

- it was shown [Yates'95] that the global minimum is achieved by the fixed-point iteration

$$p_k^{(n+1)} = \gamma_k \cdot \mathcal{I}_k(\mathbf{p}^{(n)}), \quad k = 1, 2, \ldots, K$$

Concave Interference Functions and Receive Strategies

- let $\mathcal{I}_1, \ldots, \mathcal{I}_K$ be arbitrary concave interference functions
- from the representation result, it is clear that $\mathcal{I}_k(\mathbf{p})$ has a matrix-based structure

$$\mathcal{I}_k(\mathbf{p}) = \min_{z_k \in \mathcal{Z}_k} \Big(\underbrace{\mathbf{p}^T \mathbf{v}(z_k)}_{\textit{Interference}} + \underbrace{n_k(z_k)}_{\textit{Noise}} \Big) \,, \quad k = 1, 2, \ldots, K$$

- the parameter z_k can be interpreted as a receive strategy
- for K users, we have an interference coupling matrix

$$\mathbf{V}(z) = [\mathbf{v}_1(z_1), \ldots, \mathbf{v}_K(z_K)]^T$$

Example: MMSE Beamforming

- The interference coupling $\mathbf{V}$ can depend on adaptive receive beamforming vectors $\mathbf{u}_1, \ldots, \mathbf{u}_K$, with $\|\mathbf{u}_k\| = 1$. In this case, the normalized coupling matrix $\mathbf{V}(\mathbf{u})$ is defined as

$$[\mathbf{V}(\mathbf{u})]_{kl} = \begin{cases} \frac{\mathbf{u}_k^H \mathbf{R}_l \mathbf{u}_k}{\mathbf{u}_k^H \mathbf{R}_k \mathbf{u}_k} & l \neq k \,, \\ 0 & k = l \,. \end{cases} \qquad \text{where } \mathbf{R}_l = \mathrm{E}[\mathbf{h}_l \mathbf{h}_l^H]$$

- Under this model, we have an interference function

$$\mathcal{I}_k(\mathbf{p}, \sigma_n^2) = \min_{\|\mathbf{u}_k\|=1} \left(\left[\mathbf{V}(\mathbf{u}) \cdot \mathbf{p} \right]_k + \frac{\sigma_n^2}{\mathbf{u}_k^H \mathbf{R}_k \mathbf{u}_k} \right) \qquad \text{(-1)}$$

  The function (-1) fulfills A1–A3 and is concave. For every $\mathbf{p}$, $\mathbf{u}_1, \ldots, \mathbf{u}_K$ are the respective MMSE beamformers

Example: Zeroforcing beamforming

- Under a different normalization $\mathbf{u}_k^H \mathbf{R}_k \mathbf{u}_k = 1$ we have

$$\mathcal{I}_k(\mathbf{p}, \sigma_n^2) = \min_{\mathbf{u}_k^H \mathbf{R}_k \mathbf{u}_k = 1} \big[\mathbf{V}(\mathbf{u}) \cdot \mathbf{p}\big]_k + \sigma_n^2 \cdot \|\mathbf{u}_k\|_2 \,. \qquad (0)$$

  Assuming that K is less or equal to the number of antennas, we can introduce the constraint $\mathbf{u}_k^H \mathbf{h}_l = 0$, $l \neq k$.

$$\mathcal{I}_k(\mathbf{p}, \sigma_n^2) = \min_{\substack{\mathbf{u}_k^H \mathbf{R}_k \mathbf{u}_k = 1 \\ \mathbf{u}_k^H \mathbf{h}_l = 0, l \neq k}} \|\mathbf{u}_k\|_2 \cdot \sigma_n^2 \,. \qquad (1)$$

  This is solved by the well-known least squares 'zeroforcer'.

- The function (1) is a concave interference function (though a trivial one since $\mathcal{I}_k(\mathbf{p}, \sigma_n^2)$ does no longer depend on $\mathbf{p}$).

Example: Base Station Assignment

- Consider the problem of combined beamforming and base station assignment [(**?**; **?**; **?**; **?**)].
- The kth user is received by a base station with index $b_k \in \mathcal{B}_k$.

$$\mathcal{I}_k(\mathbf{p}, \sigma_n^2) = \min_{b_k \in \mathcal{B}_k} \left(\min_{\mathbf{u}_k : \|\mathbf{u}_k\| = 1} \frac{\mathbf{u}_k^H \left(\sum_{l \neq k} p_l \mathbf{R}_l^{(b_k)} + \sigma_n^2 \mathbf{I} \right) \mathbf{u}_k}{\mathbf{u}_k^H \mathbf{R}_k^{(b_k)} \mathbf{u}_k} \right) .$$

This is a concave interference function which fulfills A1–A3.

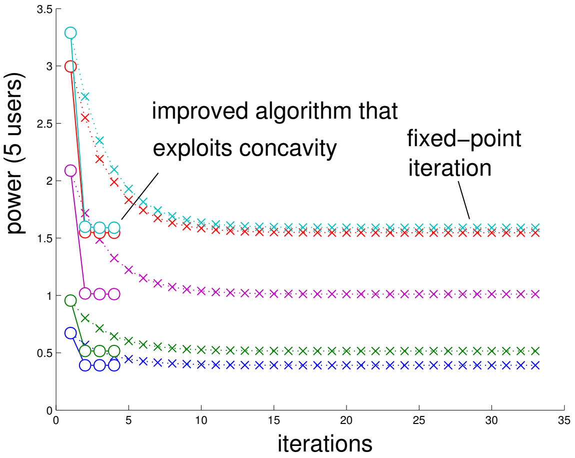
Proposed Matrix-Based Iteration

By exploiting the special structure of concave interference functions, a new iteration is obtained:

Alternating optimization of receive strategies $z^{(n)}$ and power allocation $\mathbf{p}^{(n)}$

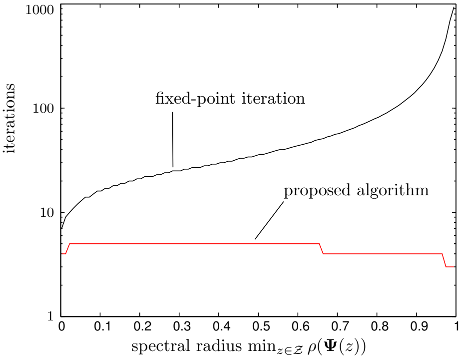
1. $z_k^{(n)} = \arg\min_{z_k \in \mathcal{Z}_k} \Big[\mathbf{V}(z)\mathbf{p}^{(n)} + \mathbf{n}(z)\Big]_k , \quad k \in \{1, 2, \ldots, K\}$
2. $\mathbf{p}^{(n+1)} = (\mathbf{I} - \mathbf{\Gamma}\mathbf{V}(z^{(n)}))^{-1} \cdot \mathbf{\Gamma}\mathbf{n}(z^{(n)})$

Convergence Behavior

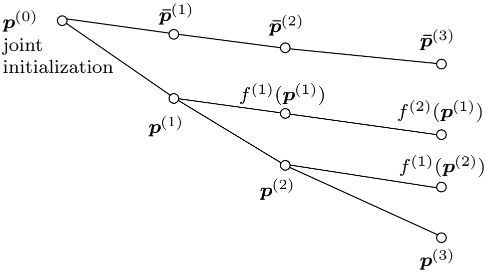


Required Iteration Steps vs. System Load

- the convergence behavior of the proposed iteration is almost independent of the system load



Direct Step-by-Step Comparison

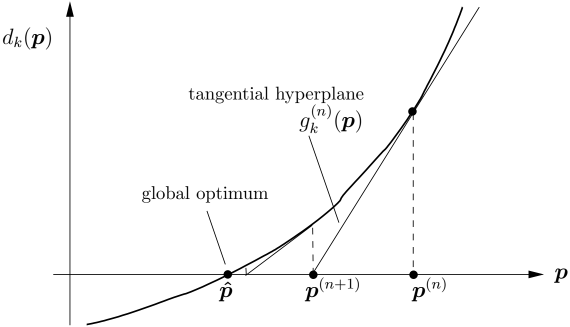


- fixed-point: $\bar{\mathbf{p}}^{(n)}$
- matrix-based iteration: $\mathbf{p}^{(n)}$

- the proposed iteration has the following advantages
  - step-wise better than the fixed point iteration, $\bar{\mathbf{p}}^{(n)} \geq \mathbf{p}^{(n)}$
  - achieves the SI(N)R targets $\boldsymbol{\Gamma}$ in each step
  - componentwise monotonicity
- both iterations converge to the global optimum of the power minimization problem

Problem Reformulation

- approach: introduce auxilliary function $\mathbf{d}(\mathbf{p}) = \mathbf{p} - \mathbf{\Gamma}\mathcal{I}(\mathbf{p})$
- the global optimum of the power minimization problem is completely characterized by $\mathbf{d}(\mathbf{p}) = 0$ (fixed-point)



- framework can be extended to non-smooth functions $\mathbf{d}(\mathbf{p})$

Super-Linear Convergence

Theorem

Let $\mathbf{p}^{(0)}$ be an arbitrary feasible initialization, then the new algorithm has super-linear convergence

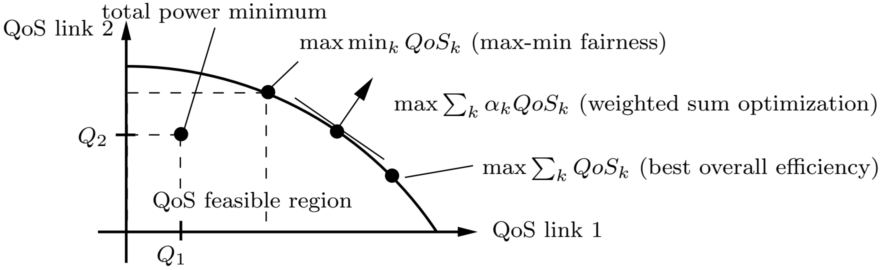
$$\lim_{n \to \infty} \frac{\|\mathbf{p}^{(n+1)} - \mathbf{p}^*\|}{\|\mathbf{p}^{(n)} - \mathbf{p}^*\|} = 0$$

- quadratic convergence is achieved for the typical case of semi-smooth interference functions ($C_1^{2^n}$ compared to C_2^n)
- the fixed-point iteration achieves only linear convergence in general

Outline

Resource Allocation

- can be regarded as the search for an "optimal" operating point in the quality-of-service (QoS) feasible region

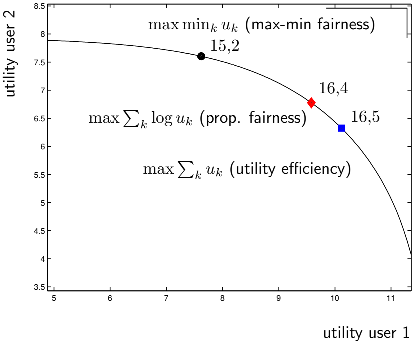


- Here, QoS stands for some performance measure which still needs to be specified

Proportional Fairness

- [Kelly'98]: The proportionally fair equilibrium $\hat{\mathbf{u}}$ is the one, at which the difference to any other utility vector $\mathbf{u} \in \mathcal{U}$ measured in the aggregated proportional change $\sum_k (u_k - \hat{u}_k)/\hat{u}_k$ is non-positive. This operating point can be found by solving

$$\max_{\mathbf{u} \in \mathcal{U}} \sum_{k=1}^{K} \log u_k$$



QoS Model for Wireless Systems

- signal-to-interference ratio

$$\mathsf{SIR}(\mathbf{p}) = \frac{p_k}{\mathcal{I}_k(\mathbf{p})}$$

- the QoS is a strictly monotonic function of the SIR

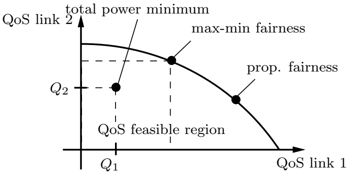
$$QoS(\mathbf{p}) = \phi\big(\mathsf{SIR}(\mathbf{p})\big)$$

examples:		
	$\phi(x) = x$	SIR
	$\phi(x) = \log(x)$	SIR in dB
	$\phi(x) = 1/(1+x)$	Min. Mean Squared Error (MMSE)
	$\phi(x) = x^{-\alpha}$	BER slope, diversity order α
	$\phi(x) = \log(1+x)$	capacity for Gaussian signals
	$\dots$	

Some Examplary Resource Allocation Problems

- max-min fairness

$$\sup_{\mathbf{p}>0}\left(\min_k \frac{p_k}{\mathcal{I}_k(\mathbf{p})}\right)$$



- sum-power minimization

$$\inf_{\mathbf{p}>0}\sum_{k=1}^{K} p_k \quad \text{s.t. } \frac{p_k}{\mathcal{I}_k(\mathbf{p})} \geq \gamma_k, \ \forall k \ ,$$

- proportional fairness [Kelly'97]

$$\inf_{\mathbf{p}>0}\sum_{k=1}^{K} \log \frac{\mathcal{I}_k(\mathbf{p})}{p_k} \ .$$

$\Rightarrow$ efficient algorithmic solutions exist for certain types of interference functions $\mathcal{I}_k(\mathbf{p})$

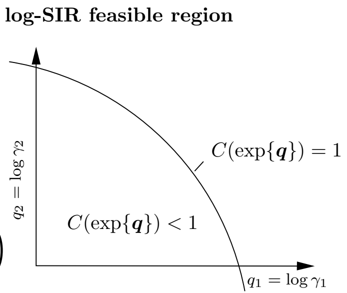
Feasible QoS Region for Log-Convex Interference Functions

- the log-SIR feasible region is

$$\mathcal{Q} = \{\mathbf{q} \in \mathbb{R}^K : C(\exp \mathbf{q}) \leq 1\}$$

where

$$C(\exp \mathbf{q}) = \inf_{\mathbf{p}>0} \Big(\max_{1 \leq k \leq K} \frac{\exp(q_k)\mathcal{I}_k(\mathbf{p})}{p_k} \Big)$$



- if $\mathcal{I}_1, \ldots, \mathcal{I}_K$ are log-convex, then $C(\exp \mathbf{q})$ is a log-convex interference function

➠ the log-SIR region is a convex set

Extension to Other QoS Measures

- Let γ_k be the inverse function of ϕ_k, then $\gamma_k := \gamma_k(q_k)$ is the minimum SIR level needed to achieve the target q_k
- assume log-convex interference functions $\mathcal{I}_1, \ldots, \mathcal{I}_K$. The QoS region is convex for all mappings $QoS = \phi(\text{SIR})$, for which the inverse function $\gamma_k(QoS_k)$ is log-convex. Examples:
    - capacity in the high SNR regime: $\phi(\text{SIR}) = \alpha \log(\text{SIR})$, with $\alpha \in \mathbb{R}$.
    - BER slope approximation: $\phi(\text{SIR}) = \text{SIR}^{-\alpha}$, for diversity order $\alpha \geq 0$.
    - ...

Proportional Fairness for Log-Convex Interf. Functions

- for utilities $u_k = p_k / \mathcal{I}_k(\mathbf{p})$ the problem of proportional fairness can be rewritten as

$$PF(\mathcal{I}) = \inf_{\mathbf{p}>0} \sum_{k=1}^{K} \log \frac{\mathcal{I}_k(\mathbf{p})}{p_k}$$

- if $\mathcal{I}_1, \ldots, \mathcal{I}_K$ are log-convex interference functions, then this is a convex optimization problem

Weighted Proportional Fairness

- consider weighting factors $\alpha_1, \ldots, \alpha_K$. The weighted proportionally fair optimum is

$$\inf_{\mathbf{s} \in \mathbb{R}^K} \sum_{k=1}^{K} \alpha_k \, g\big(\mathcal{I}_k(\mathrm{e}^{\mathbf{s}})/\mathrm{e}^{s_k}\big) \quad \text{s.t.} \;\; \|\mathrm{e}^{\mathbf{s}}\|_1 \leq P_{max}$$

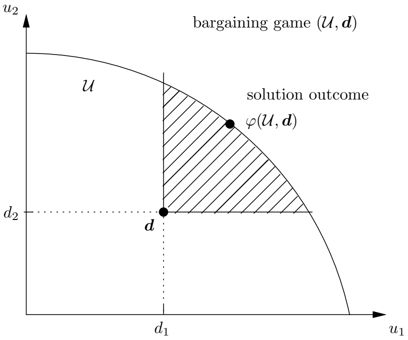
Theorem

Let $\mathcal{I}_k(\mathrm{e}^{\mathbf{s}})$ be log-convex $\forall k$ and g monotonic increasing. Then the problem is convex if and only if $g(\mathrm{e}^x)$ is convex on $\mathbb{R}^K$

Outline

A Game Theoretic View: Cooperative Bargaining

- the service qualities of all K users is modeled by a utility vector $\mathbf{u} = [u_1, \ldots, u_k]$, chosen from a region $\mathcal{U}$
- The players try to reach an unanimous agreement on some outcome $\mathbf{u} \geq \mathbf{d}$
- If they fail, the disagreement outcome or disagreement point $\mathbf{d}$ results.
- the solution outcome $\varphi(\mathcal{U}, \mathbf{d})$ is the operating point of the system



Axiomatic Framework for Symmetric Nash Bargaining

WPO Weak Pareto Optimality: The players should not be able to collectively improve upon the solution outcome.

IIA Independence of Irrelevant Alternatives: If the feasible set shrinks but the solution outcome remains feasible, then the solution outcome for the smaller feasible set should be the same point.

SYM Symmetry: If the region is symmetric, then the outcome does only depend on the employed strategies and not on the identities of the users. Axiom SYM basically means that all users have the same priorities.

STC Scale Transformation Covariance: The optimization strategy is invariant with respect to a component-wise scaling of the region.

The Symmetric Nash Bargaining Solution

Let $\mathcal{U}$ be convex and $\mathbf{u}' \leq \mathbf{u}$, $\mathbf{u} \in \mathcal{U}$ implies $\mathbf{u}' \in \mathcal{U}$. Then, the unique outcome fulfilling the axioms WPO, IIA, SYM, STC, is called symmetric Nash bargaining solution (SNBS).

- SNBS is equivalent to the solution of

$$\max_{\{\mathbf{u} \in \mathcal{U} : \mathbf{u} > \mathbf{d}\}} \prod_{k=1}^{K} (u_k - d_k)$$

- we can assume $\mathbf{d} = 0$, thus

$$\max_{\mathbf{u} \in \mathcal{U}} \prod_{k=1}^{K} u_k$$

Equivalence between SNBS and Proportional Fairness

- the product optimization approach is equivalent to proportional fairness [Kelly'98]

$$\hat{\mathbf{u}} = \arg\max_{\mathbf{u}\in\mathcal{U}} \prod_{k=1}^{K} u_k = \arg\max_{\mathbf{u}\in\mathcal{U}} \log \prod_{k=1}^{K} u_k = \arg\max_{\mathbf{u}\in\mathcal{U}} \sum_{k=1}^{K} \log u_k$$

- if the region $\mathcal{U}$ is convex and monotonic, then symmetric Nash bargaining and proportional fairness are equivalent

Representation of the Convex Set $\mathcal{U}$

Every convex utility-region $\mathcal{U}$ can be represented as a sub-level set of a convex function

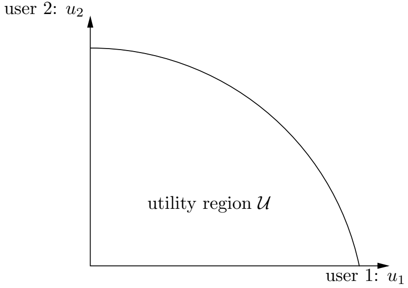
$$\mathcal{U} = \{\mathbf{u} \in \mathbb{R}_+^K : \mathcal{I}_{\mathcal{U}_1}(\mathbf{u}) \leq 1\}$$

where

$$\mathcal{I}_{\mathcal{U}_1}(\mathbf{p}) = \max_{\mathbf{w} \in \mathcal{U}_1} \sum_{k=1}^{K} w_k \cdot p_k, \quad \mathbf{p} > 0$$

and $\mathcal{U}_1 = \{\mathbf{u} \in \mathbb{R}_+^K : \mathcal{I}_{\mathcal{U}}(\mathbf{u}) \leq 1\}$

and $\mathcal{I}_{\mathcal{U}}(\mathbf{p}) = \max_{\mathbf{w} \in \mathcal{U}} \sum_{k=1}^{K} w_k \cdot p_k, \quad \mathbf{p} > 0$



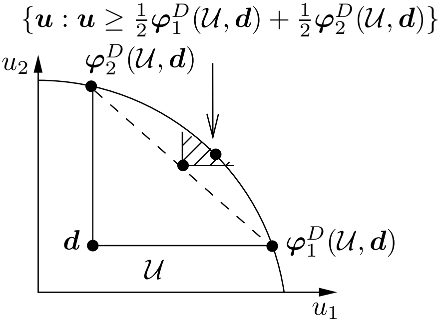
Using the Representation of Log-Convex Interf. Functions

- the product optimum over $\mathcal{U}$ is equivalently obtained by computing $f_{\mathcal{I}_\mathcal{U}}(\mathbf{w})$, with $\mathbf{w} = [1, 1, \ldots, 1]$

$$\max_{\mathbf{u}\in\mathcal{U}} \prod_{k=1}^{K} u_k = \frac{1}{f_{\mathcal{I}_\mathcal{U}}(\mathbf{w})} .$$

- this shows that the product optimization problem is closely linked with the log-convex structure of $\mathcal{U}$

Mid-Point Dominance



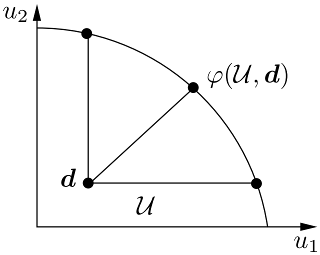
- $\varphi_1^D(\mathcal{U},\mathbf{d})$ is the dictatorial solution for user 1
- $\varphi_2^D(\mathcal{U},\mathbf{d})$ is the dictatorial solution for user 2

the mid-point dominance axiom requires

$$\varphi(\mathcal{U},\mathbf{d}) \geq \frac{1}{K}\sum_{k=1}^{K}\varphi_k^D(\mathcal{U},\mathbf{d})$$

➠ minimal amount of cooperation between users

Disagreement Point Convexity



the "disagreement point convexity" axiom requires:

$$\mathbf{d}(\mu) = (1-\mu)\mathbf{d} + \mu\varphi(\mathcal{U}, \mathbf{d}) \implies \varphi(\mathcal{U}, \mathbf{d}(\mu)) = \varphi(\mathcal{U}, \mathbf{d}) \qquad (0 < \mu < 1)$$

➠ this models the impact of the user requirements

Example of a Non-Standard Characterization of the SNBS

Theorem

the symmetric Nash bargaining solution is the only solution that satisfies the axioms "mid-point dominance" and "disagreement point convexity"

advantage: we can analyze the impact of user cooperation and user requirements

Outline

Conclusions

- the framework of interference functions is applicable to different areas
  - physical layer design
  - medium access control
  - resource allocation and utility optimization for wireless systems
  - how to operate a wireless system
- many interesting open questions