Unequal Error Protection Turbo and LDPC codes

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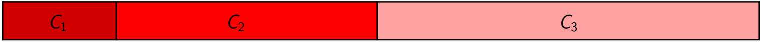
Overview

- Unequal Error Protection (UEP)
- UEP convolutional and Turbo codes
  - Pruning vs. puncturing
- Bit-irregular UEP LDPC codes
  - Irregular bit-node profile
  - Behaviour and optimisation target
  - Algorithm
- Bit-irregular UEP LDPC codes for higher order constellations
  - Motivation
  - Extended density evolution
- Summary

Unequal Error Protection (UEP)

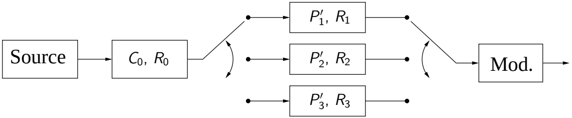
- Data of different importance from source encoder



- Multimedia:
  - Header
  - DC components, magnitude information
  - AC components, high resolution data, position information
- Different protection classes/levels provided by
  - Physical layer: modulation, bit loading, power loading, ...
  - Network layer: network coding, protocols, ...
  - Channel coding: variable rate codes, local properties, ...

Conventional UEP Convolutional Codes

- Goal: a code family with several code rates $R_i = \frac{k_i}{n_i}$
- Well-known: Puncturing
- Increased code rate by excluding code bits from transmission



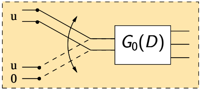
Decoding

- Punctured positions are known to the receiver
- Content is unknown, 0 and 1 are assumed equally likely
- No contribution to decoding

What about reducing the code rate?

New Approach: Pruning

- Reduced code rate by inserting known bits into the info sequence
- Pruning pattern: Insert bits periodically in certain patterns ($\leftrightarrow$ puncturing pattern)
- Fixing bits means pruning paths from the trellis $\longrightarrow$ sub-codes

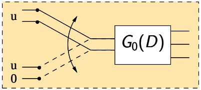


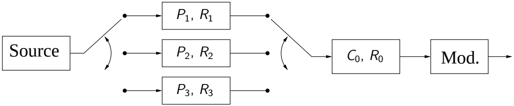
Decoding

- Receiver knows pruned positions and their content
- A-priori knowledge of pruned positions is infinite!
- Turbo codes: Increased extrinsic information!

New Approach: Pruning

- Reduced code rate by inserting known bits into the info sequence
- Pruning pattern: Insert bits periodically in certain patterns (↔ puncturing pattern)
- Fixing bits means pruning paths from the trellis → sub-codes





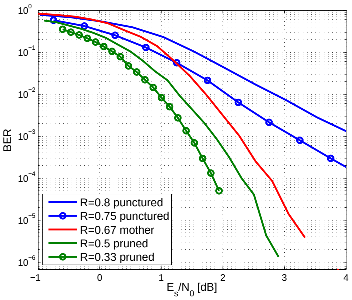
Decoding

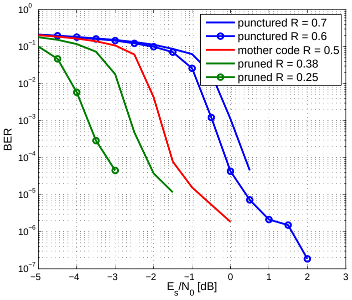
- Receiver knows pruned positions and their content
- A-priori knowledge of pruned positions is infinite!
- Turbo codes: Increased extrinsic information!

Time-Variant Pruning - Results

- Convolutional (NSC) mother code, $R = 2/3$, N $= 200$
- Punctured: $R = 3/4$ and $R = 4/5$
- Pruned: $R = 1/2$ and $R = 1/3$

- Turbo code with (RSC) mother code, $R = 0.5$, N $= 1000$
- Punctured: $R = 0.6$ and $R = 0.7$
- Pruned: $R = 0.25$ and $R = 0.38$





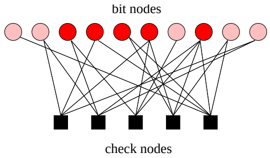
Bit-Irregular UEP LDPC Codes

LDPC codes

- Block codes with sparse parity-check matrix H
- Irregular bit-node profile $\tilde{\lambda}(x)$
- Group bit nodes into protection classes
- Define and optimise bit-node profile for each class $\rightarrow \tilde{\lambda}^{(k)}(x)$

$$\begin{aligned}\tilde{\lambda}(x) &= \tilde{\lambda}_2 \cdot x + \tilde{\lambda}_3 \cdot x^2 + \ldots \\ &= \tilde{\lambda}_2^{(1)} \cdot x + \tilde{\lambda}_2^{(2)} \cdot x + \tilde{\lambda}_3^{(1)} \cdot x^2 + \tilde{\lambda}_3^{(2)} \cdot x^2 + \ldots \\ &= \tilde{\lambda}^{(1)}(x) + \tilde{\lambda}^{(2)}(x)\end{aligned}$$

- Common check-node profile $\tilde{\rho}(x)$ assumed for all classes



Inherent and enhanced UEP of LDPC Codes

Inherent UEP

- Highly connected nodes are protected better!
- Most connected bit nodes assigned to most important class
- Maximise average connection degree of bit nodes in each class

Enhanced UEP

By density evolution, one can show that a class' error probability is minimised by

- maximising the average connection degree $d_{v_{av}}^{(k)}$ and
- maximising the minimum degree $d_{v_{min}}^{(k)}$ of its bit nodes.

Optimisation strategy

Optimisation is done class after class, most important class first!

Objective function and constraints

- Objective functions: $\max d_{v_{av}}^{(k)}$ and $\max d_{v_{min}}^{(k)}$
- Constraints:
  - $C1$ Rate constraint
  - $C2$ Proportion distribution constraints
  - $C3$ Convergence constraint
  - $C4$ Stability condition
  - $C5$ Minimum bit-node degree constraint
  - $C6$ Previous classes constraints

Used optimisation method

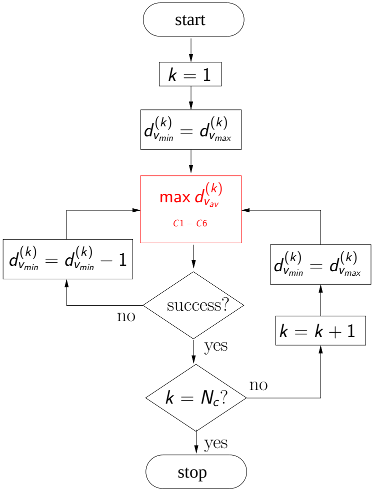
- Linear programming (LIPSOL, simplex)
- Single objective function
- Equality/inequality constraints, lower/upper bounds

Hierarchical Optimisation Algorithm

- Given parameters: σ^2 (SNR), $\tilde{\rho}(x)$, N_c, R
- For each class k, starting with the most important class
  - Initialisation $d_{v_{min}}^{(k)} = d_{v_{max}}^{(k)}$
  - While optimisation failure
    - Maximise average connection degree of class k:

      $$\max d_{v_{av}}^{(k)}$$

      fulfilling constraints $C1$-$C6$
    - $d_{v_{min}}^{(k)} = d_{v_{min}}^{(k)} - 1$
  - End
- End



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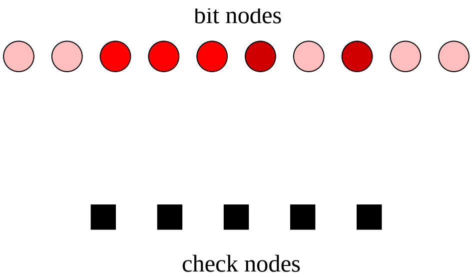

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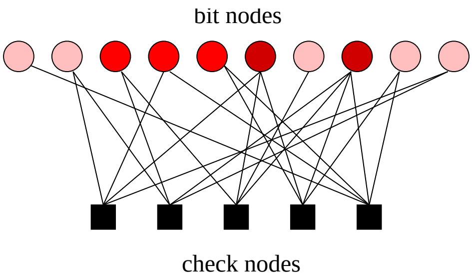

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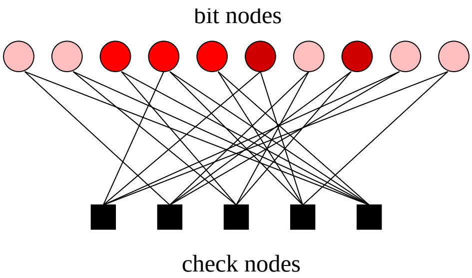

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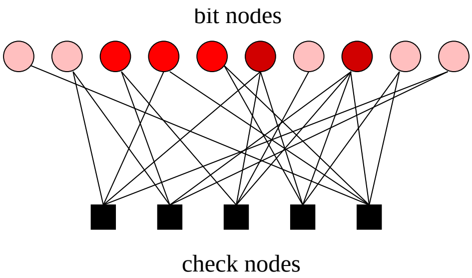

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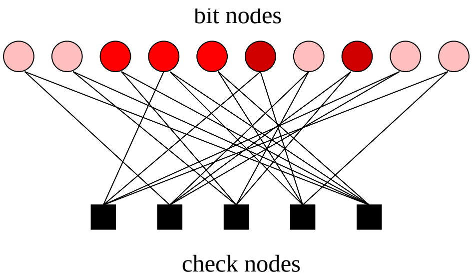

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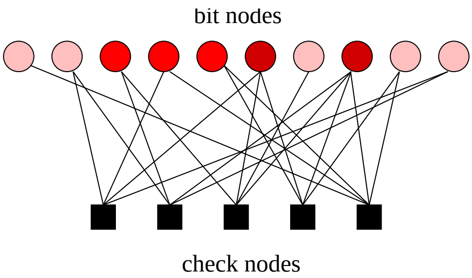

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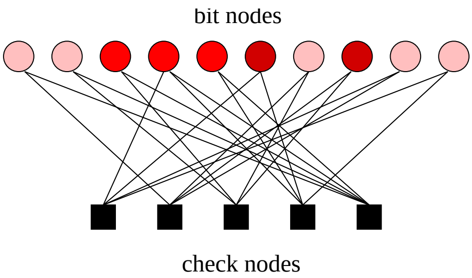

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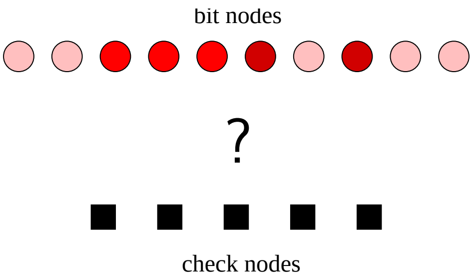

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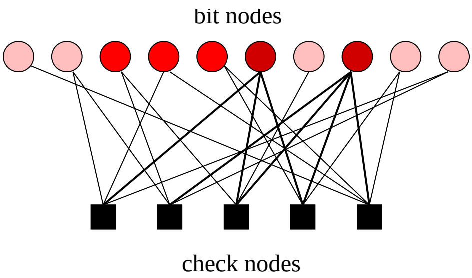

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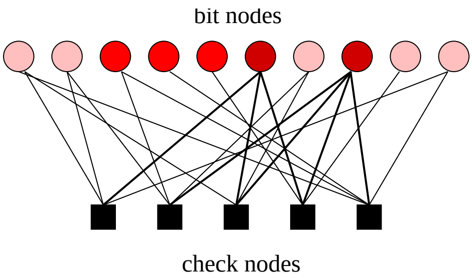

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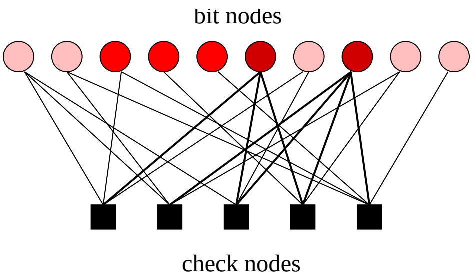

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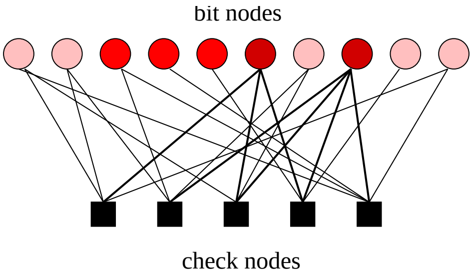

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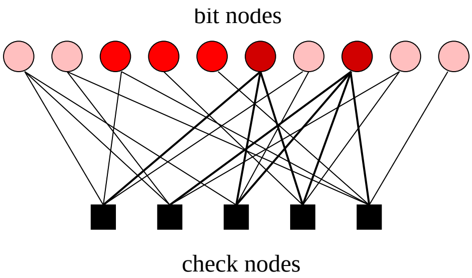

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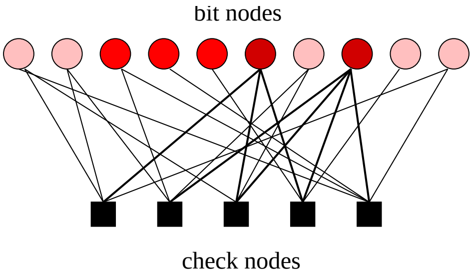

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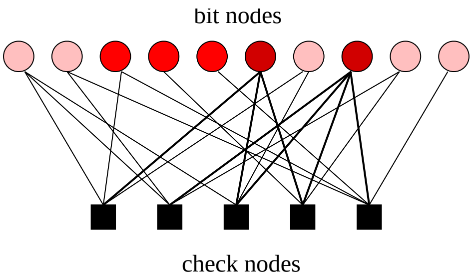

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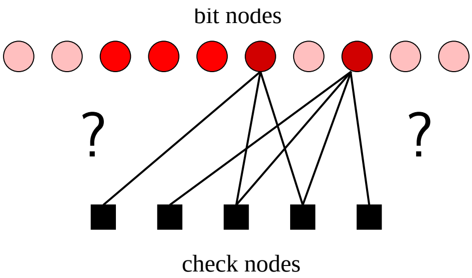

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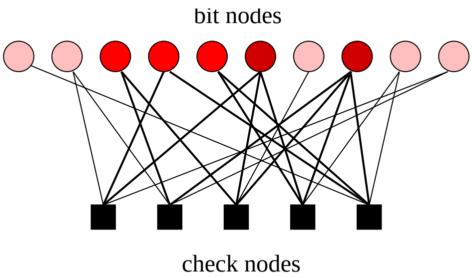

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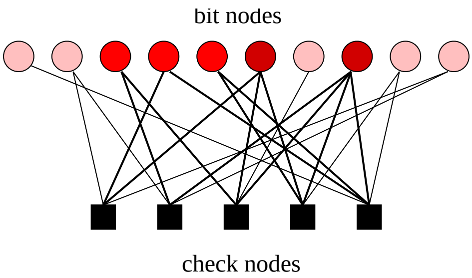

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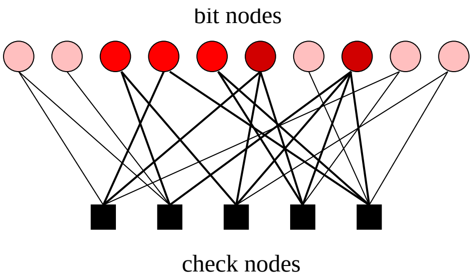

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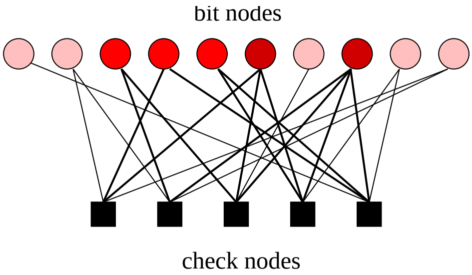

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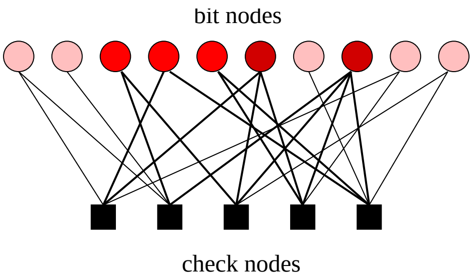

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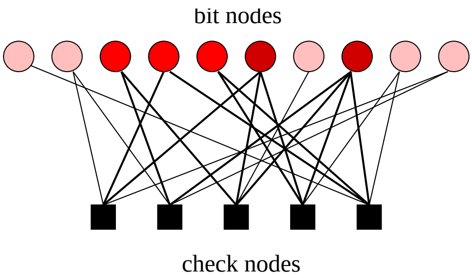

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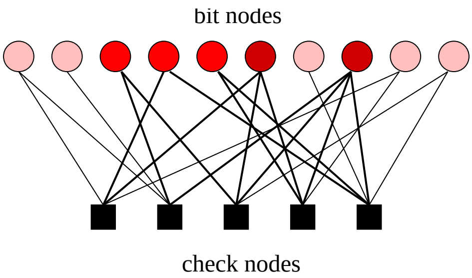

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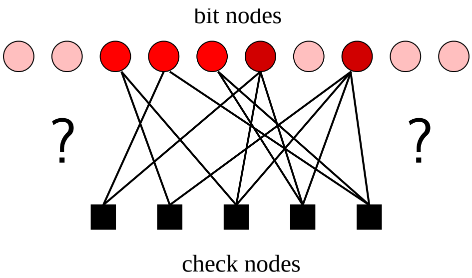

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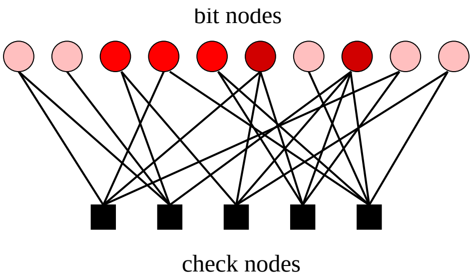
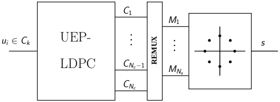


Bit-Irregular UEP LDPC Codes for Higher Order Constellations

Motivation



- Heterogeneous bit-error probabilities
- Example 8-PSK:

$$P_{b,d_0} = 1/2 \cdot P_s$$
$$P_{b,d_1} = P_{b,d_2} = 1/4 \cdot P_s$$

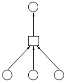
- Conventional LDPC-design assumes homogeneous channel noise variance σ^2 in density evolution
- Noise variance is not any more equal for all bits!

Tasks for improved design

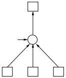
- Find a way to treat bit positions differently
- Find equivalent noise variances for the bit positions

Improved Design

Extended density evolution



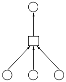
$$I_{cv}^{(l-1)} = 1 - \sum_{t=2}^{d_{c_{max}}} \rho_t \cdot J\left((t-1) \cdot J^{-1}\left(1 - I_{vc}^{(l-1)}\right)\right)$$



$$I_{vc}^{(l)} = \sum_{i=2}^{d_{v_{max}}} \lambda_i \cdot J\left(\frac{2}{\sigma^2} + (i-1) \cdot J^{-1}(I_{cv}^{(l-1)})\right)$$

Improved Design

Extended density evolution



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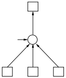
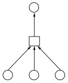


$$I_{vc}^{(l)} = \sum_{k=1}^{N_c} \sum_{i=2}^{d_{vmax}} \lambda_i^{(k)} \cdot J\left(\frac{2}{\sigma^2} + (i-1)\cdot J^{-1}(I_{cv}^{(l-1)})\right)$$

Improved Design

Extended density evolution



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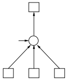
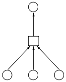


$$I_{vc}^{(l)} = \sum_{j=1}^{N_s} \sum_{k=1}^{N_c} \sum_{i=2}^{d_{v_{max}}} \lambda_i^{(k,j)} \cdot J\left(\frac{2}{\sigma_j^2} + (i-1) \cdot J^{-1}(I_{cv}^{(l-1)})\right)$$

Improved Design

Extended density evolution



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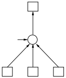

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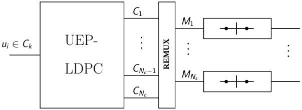
Equivalent noise variances σ_j^2

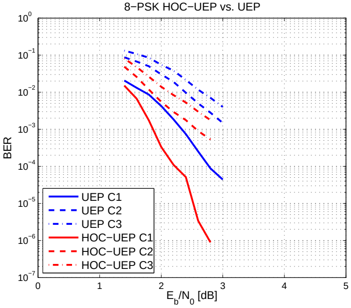
- Gray labelling: $\log_2 M$ equivalent BPSK channels
- Equivalent noise variances based on bit-error probabilities
- $\sigma^2 \overset{\text{8-PSK}}{\equiv} P_s \overset{\text{Gray}}{\longrightarrow} P_{b,d_i} \overset{\text{BPSK}}{\equiv} \sigma_j^2$

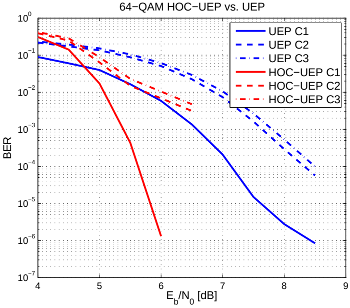


UEP LDPC codes for HOC - Results

We compare

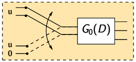
- UEP codes designed for BPSK (UEP) with
- UEP codes designed for 8-PSK/64-QAM (HOC-UEP).
- Both schemes are used with higher order constellations.
- $N_c = 3,\ n = 4096,\ R = 0.5,\ N_{it} = 20$



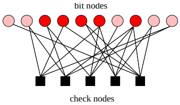


Summary

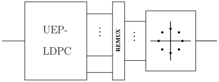
- Pruned UEP convolutional and Turbo codes
  - → Pruning through inserting zeros into the info sequence
  - → Set of BER curves in both directions



- Bit-irregular UEP LDPC codes
  - → Different bit-node profiles for protection classes
  - → Hierarchical optimisation algorithm



- UEP LDPC codes for higher order constellations
  - → Awareness of heterogeneous noise variance
  - → Extended density evolution



Thank you!