Random Matrices of Bosonic Type
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Classical settings for *random matrix* considerations are most often on the fermionic side. For example, one starts with an action of a group K_n of unitary transformations and studies its action, usually by conjugation. on relevant spaces $\mathcal{H}_n$ of Hermitian operators. Given a K_n-invariant probability density, e.g., of Gaussian type, one derives asymptotic properties of induced distributions of various spectral invariants. Many interesting results have been proved, both classically and in recent years. One key ingredient on this fermionic side is that the groups K_n are compact and it is not difficult to employ invariant Graussians and their perturbations.

On the bosonic side the difficulties arise at the very beginning where the symmetry group $G_n = Sp_{2n}(\mathbb{R})$ is non-compact. The space $\mathcal{H}_n$ of Hermitian matrices where G_n acts is well known, but only certain regions, Ω_n, in this space are appropriate for the analogous random matrix theory. In this setting we introduce a procedure which produces maximally invariant measures on Ω_n which are canonically associated to measures of fundamental importance in a certain complex analytic (resp. symplectic geometric) setting. Numerical studies for small n indicate that the associated distributions at the spectral level have the shape required by physics.

In the lecture we will sketch the setup on the fermionic side and indicate some classical results. After a brief study of the action of the symmetry group on Ω_n we will turn to the basic symplectic setting where canonical measures arise via the theory of symmetric spaces. The measures on Ω_n are produced by a direct image process.