Soap bubbles and polyhedra

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Modern Mathematics
Jacobs Univ., Bremen
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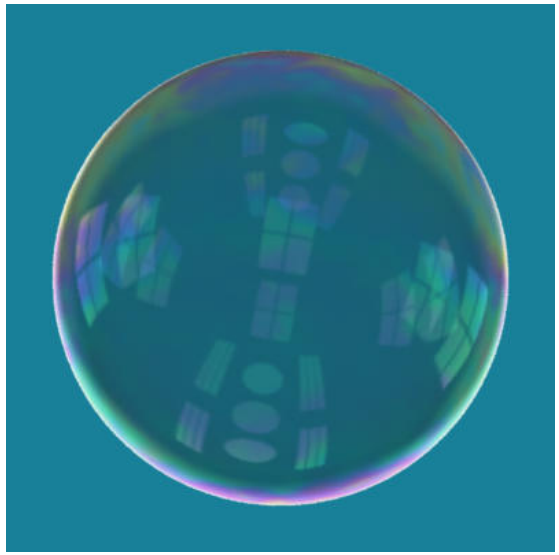
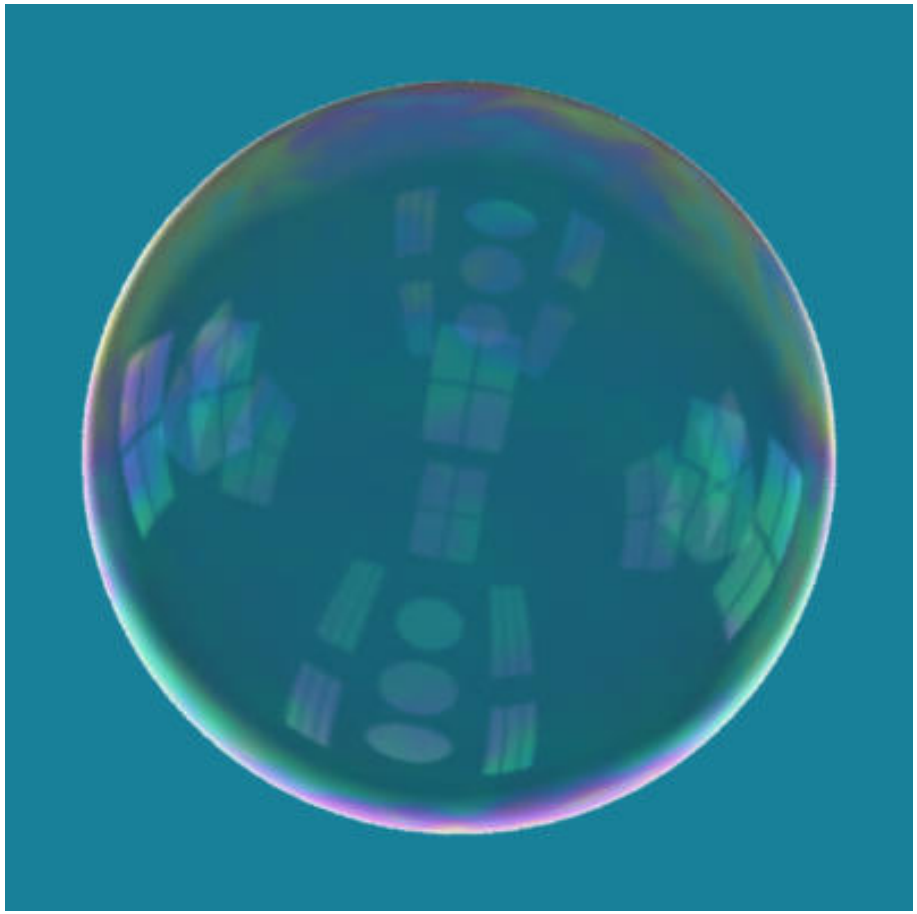


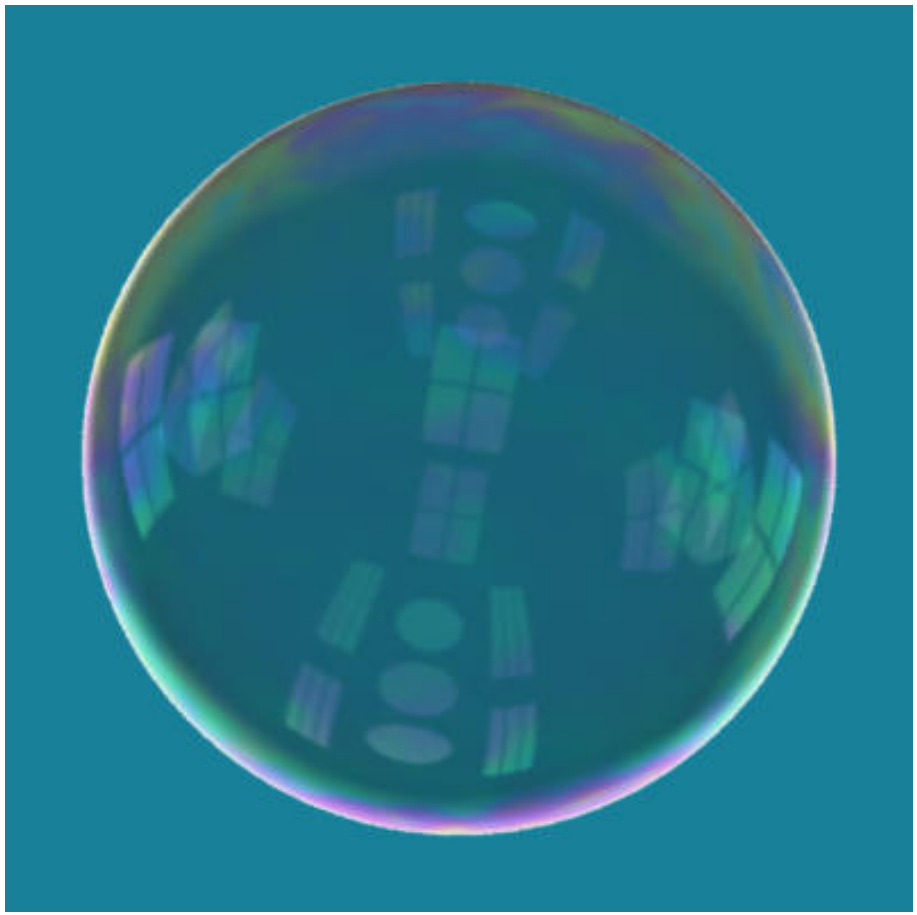
Soap bubbles

- Why are they round?



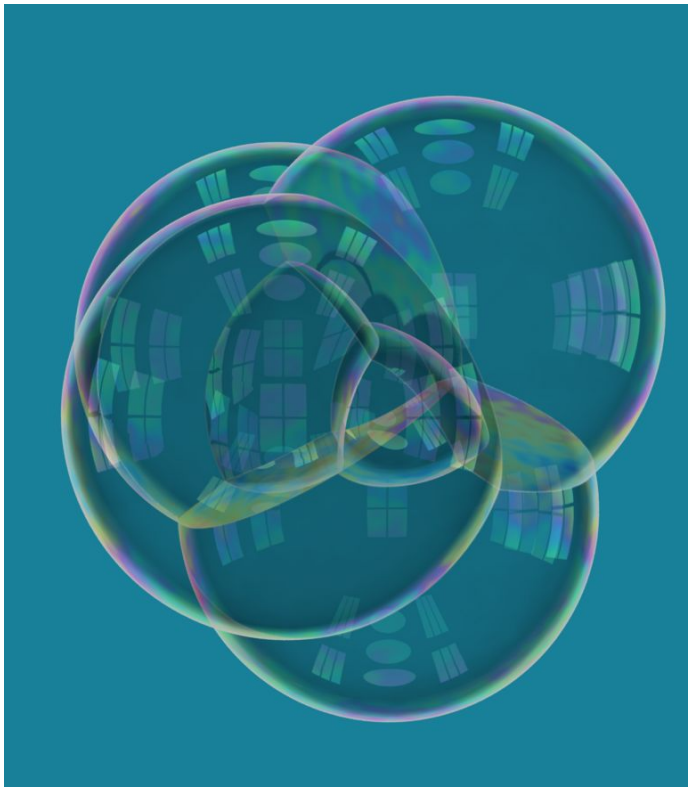
Soap bubbles

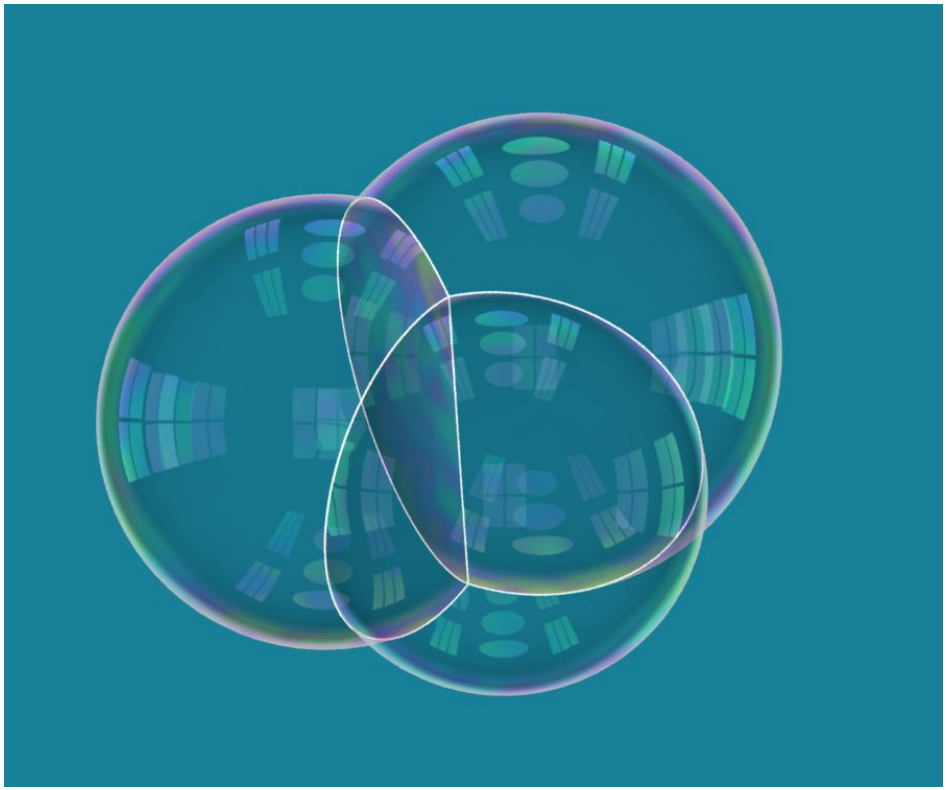
- Why are they round?
- Soap film minimizes area
- Symmetrization (Schwarz 1884)



Bubble clusters

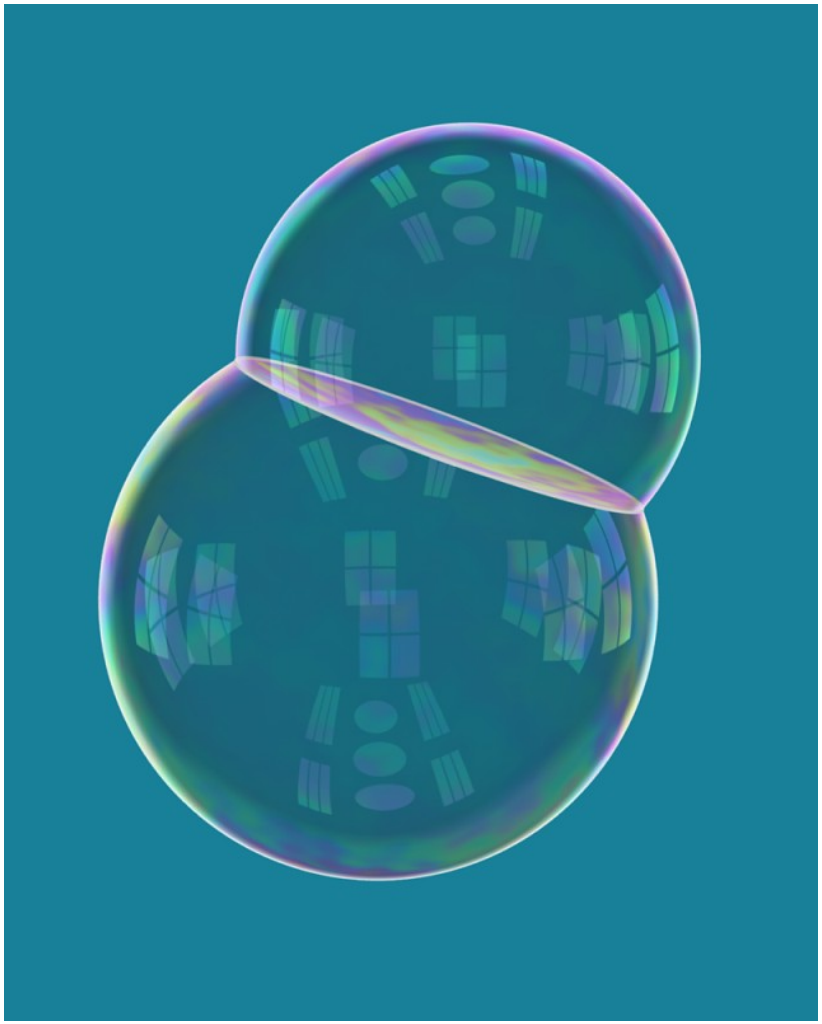
- several bubbles stuck together

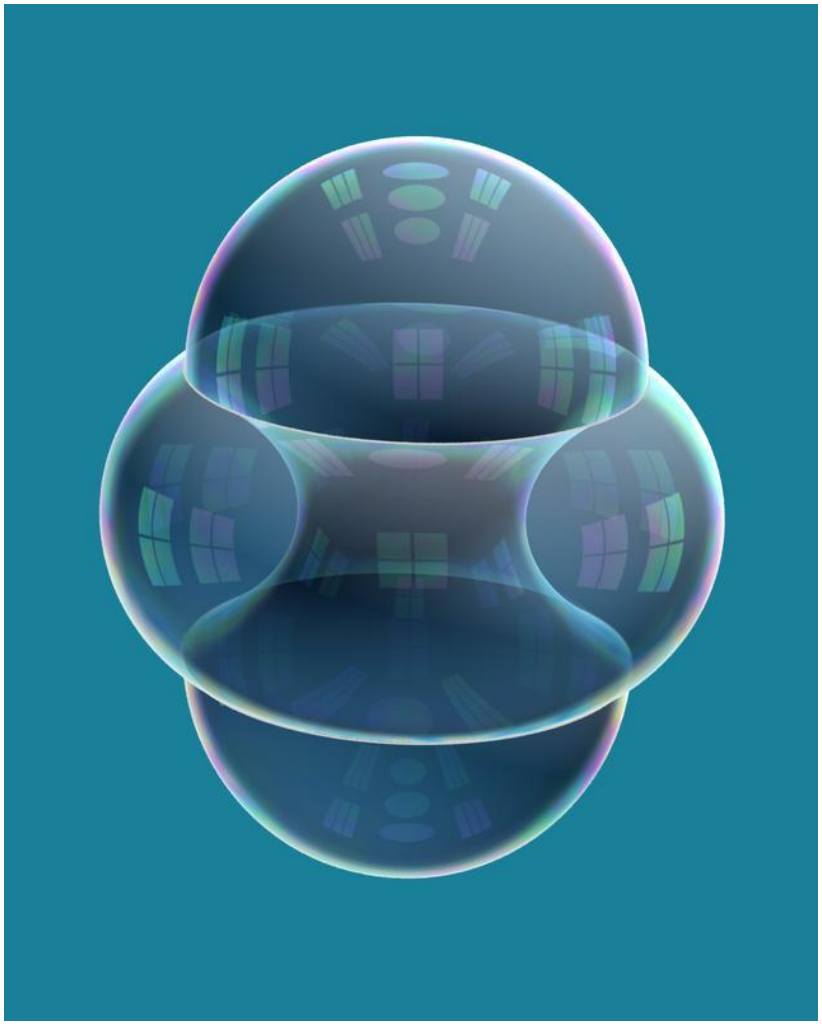




Double bubble

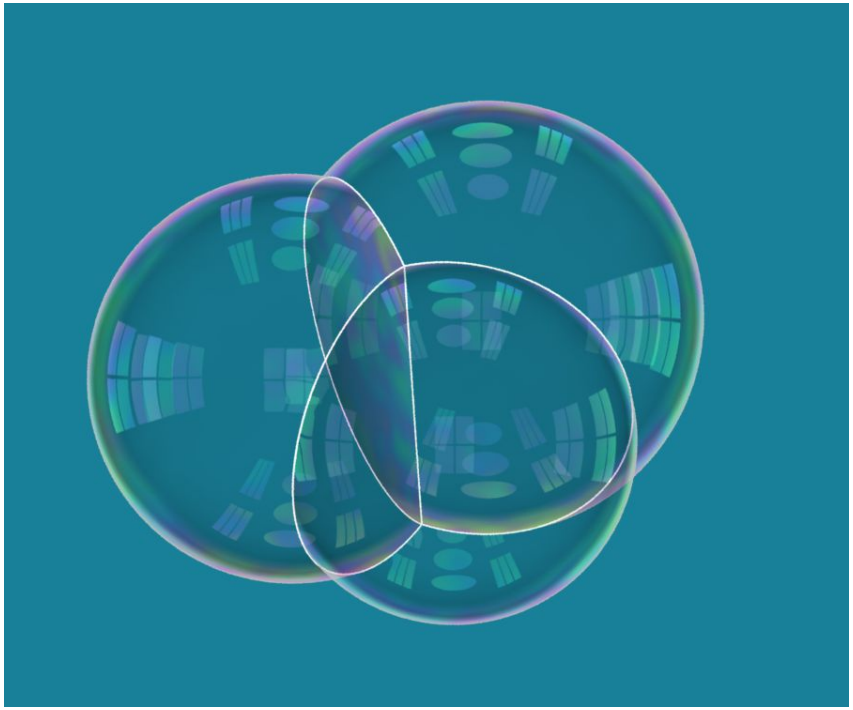
- Two bubbles with given volumes
- 1990s

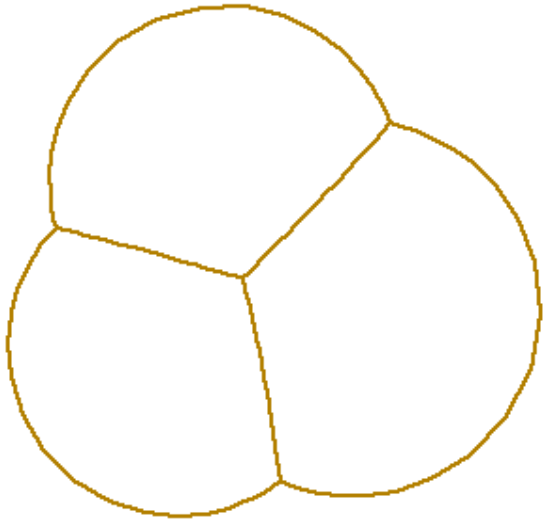




Triple bubble

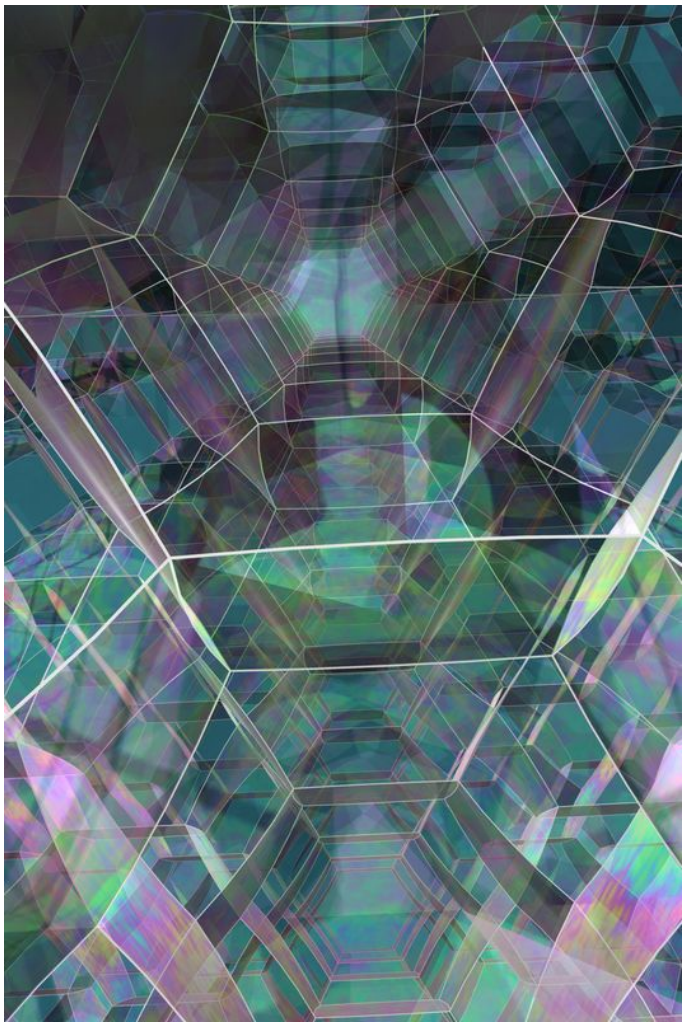
- Three bubbles with given volumes
- (2D Wichiramala 2004)

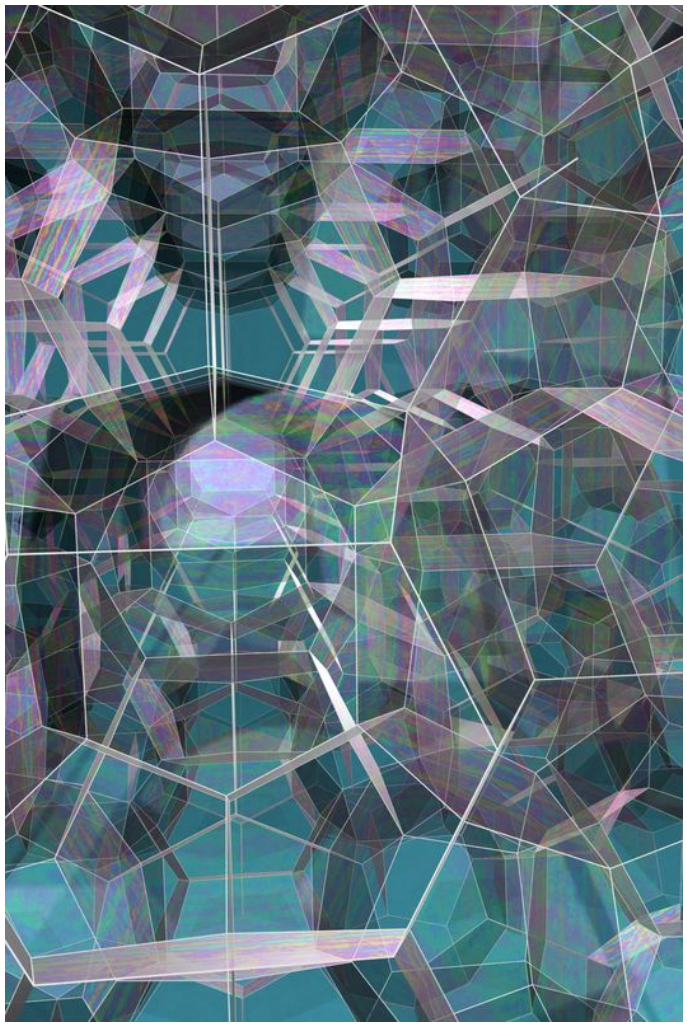




Foam

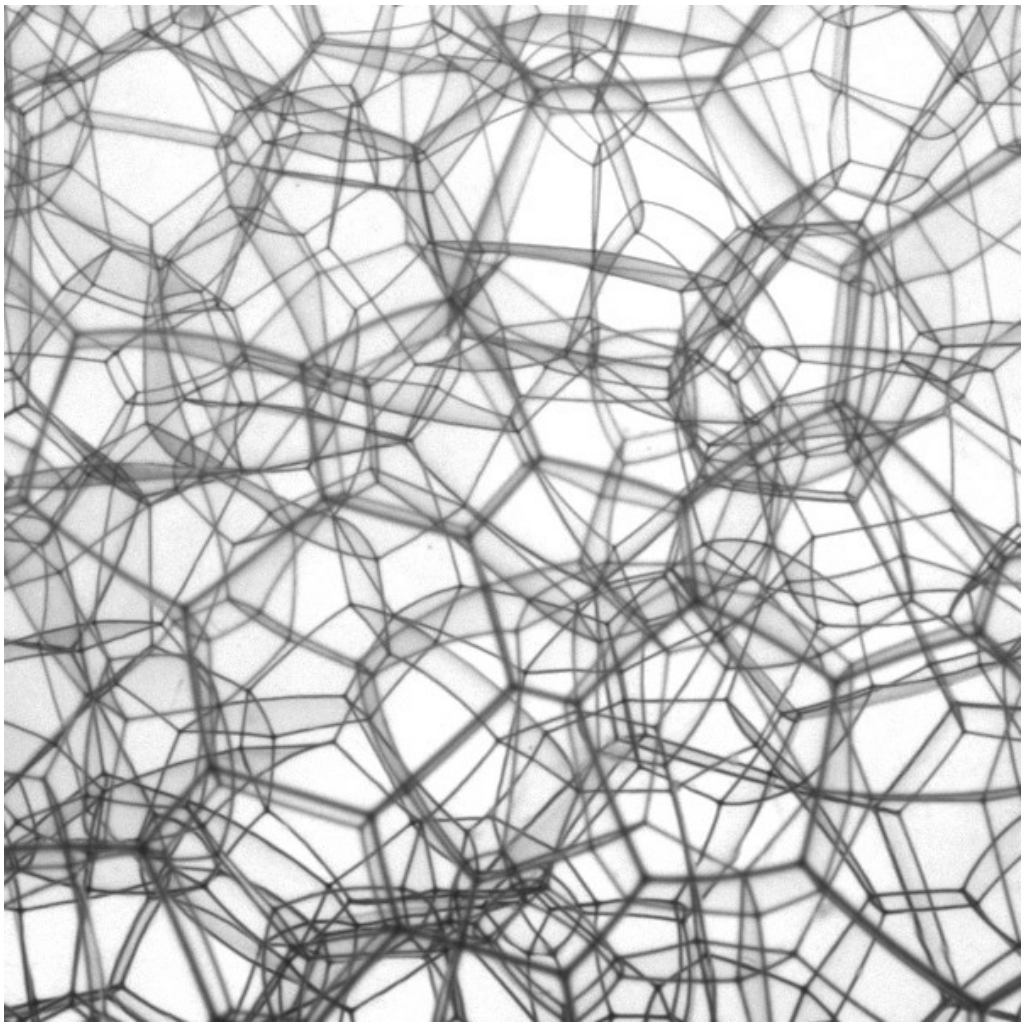
- up to 1000s of bubbles





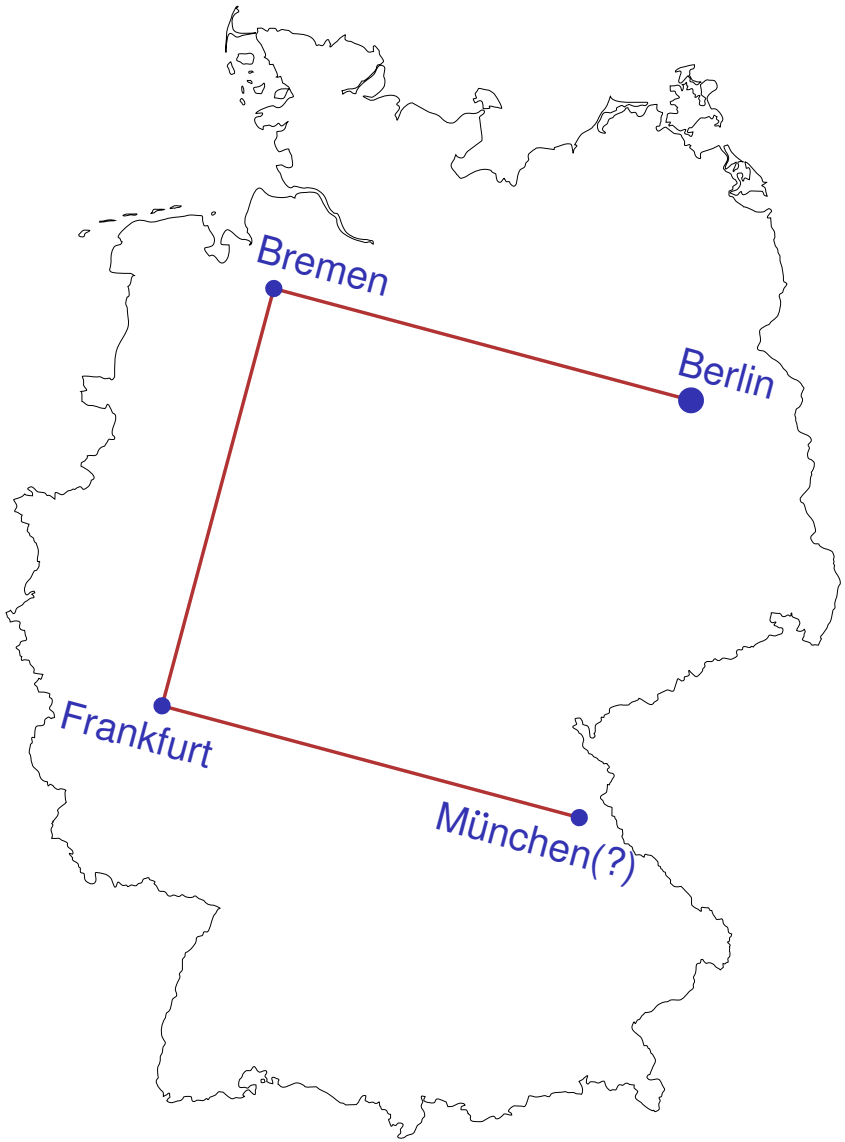
Foam

- up to 1000s of bubbles



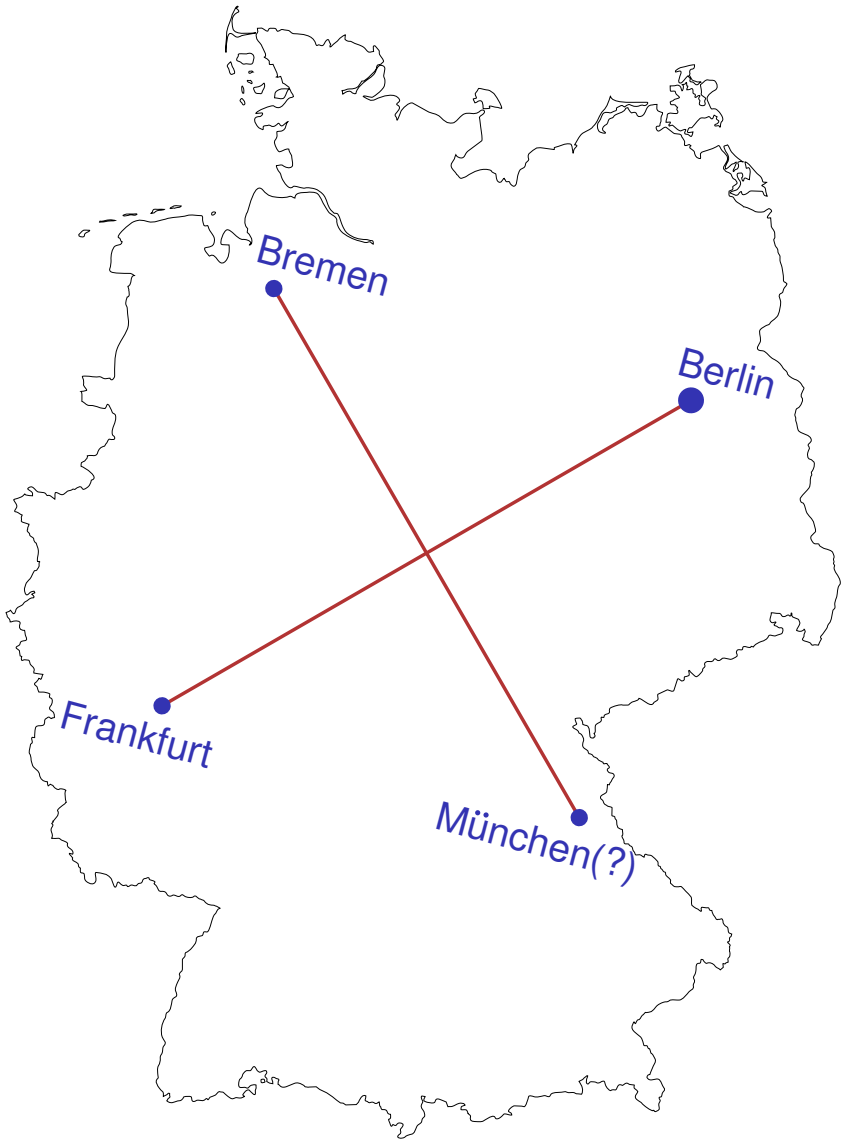
New highways

- connect four cities
- square with $s \approx 400$km
- length $3s$



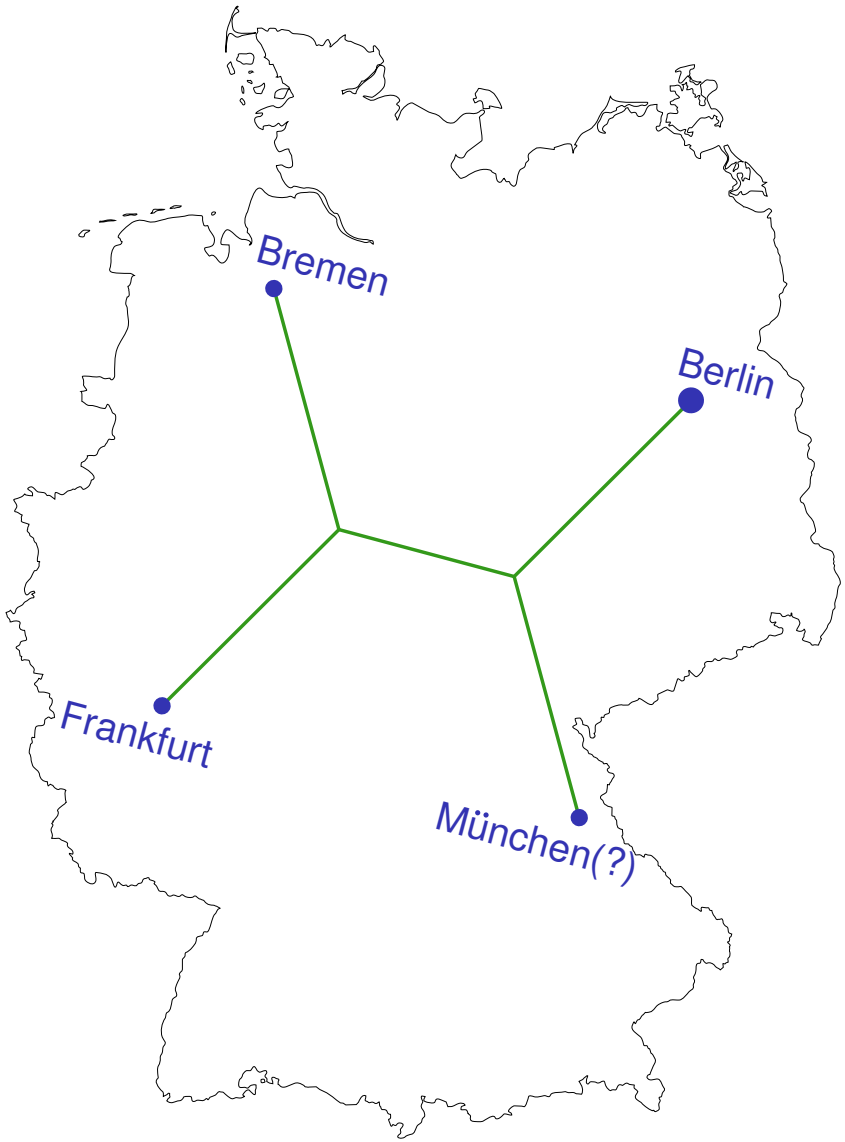
New highways

- connect four cities
- square with $s \approx 400$km
- length $2\sqrt{2}s \approx 2.83s$



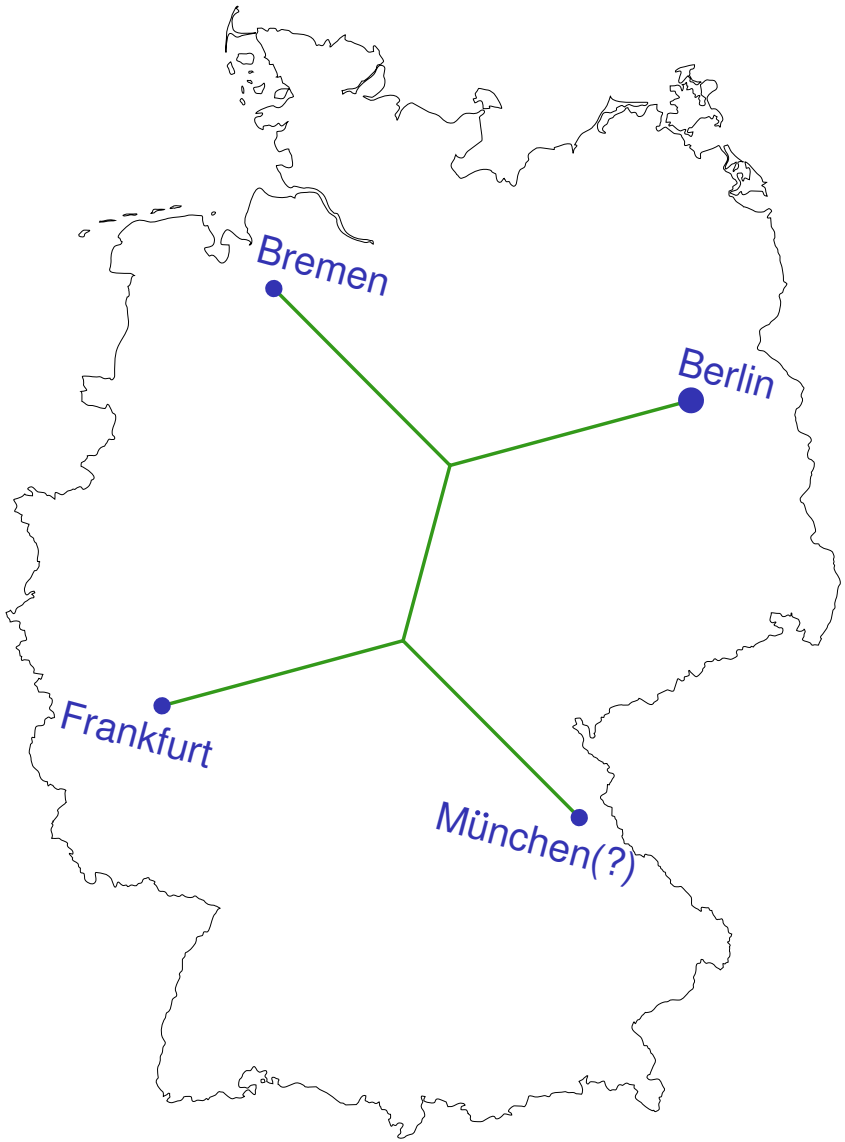
New highways

- connect four cities
- square with $s \approx 400$km
- length $(1 + \sqrt{3})s \approx 2.73s$



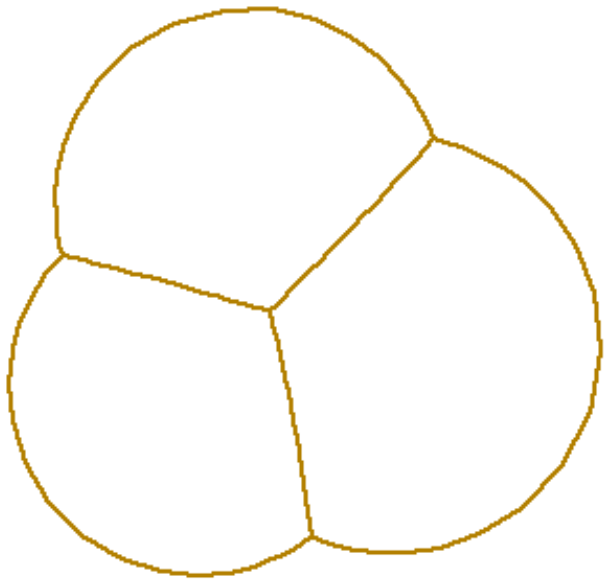
New highways

- connect four cities
- square with $s \approx 400$km
- length $(1 + \sqrt{3})s \approx 2.73s$
- **X** $\to$ 2 **Y**



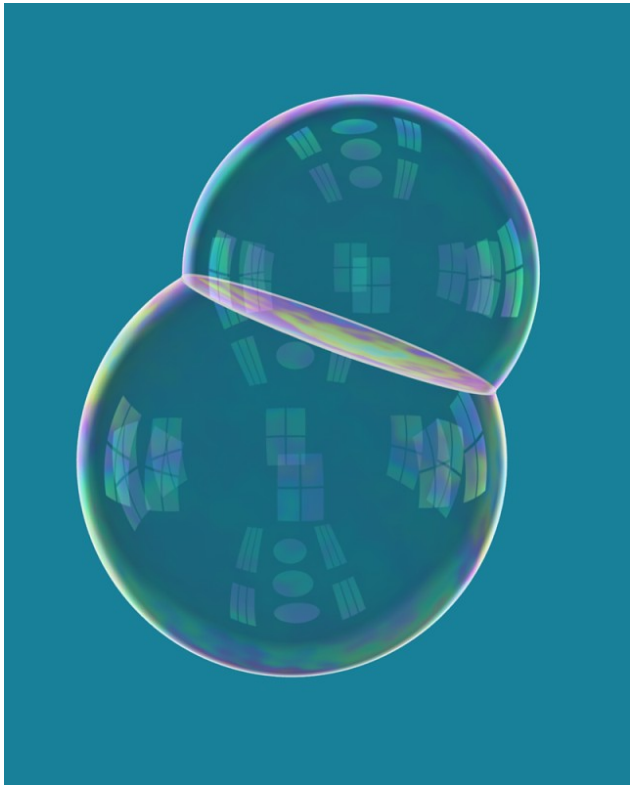
Bubble clusters in Flatland

- circular arcs
- triple junctions
- equal $120°$ angles

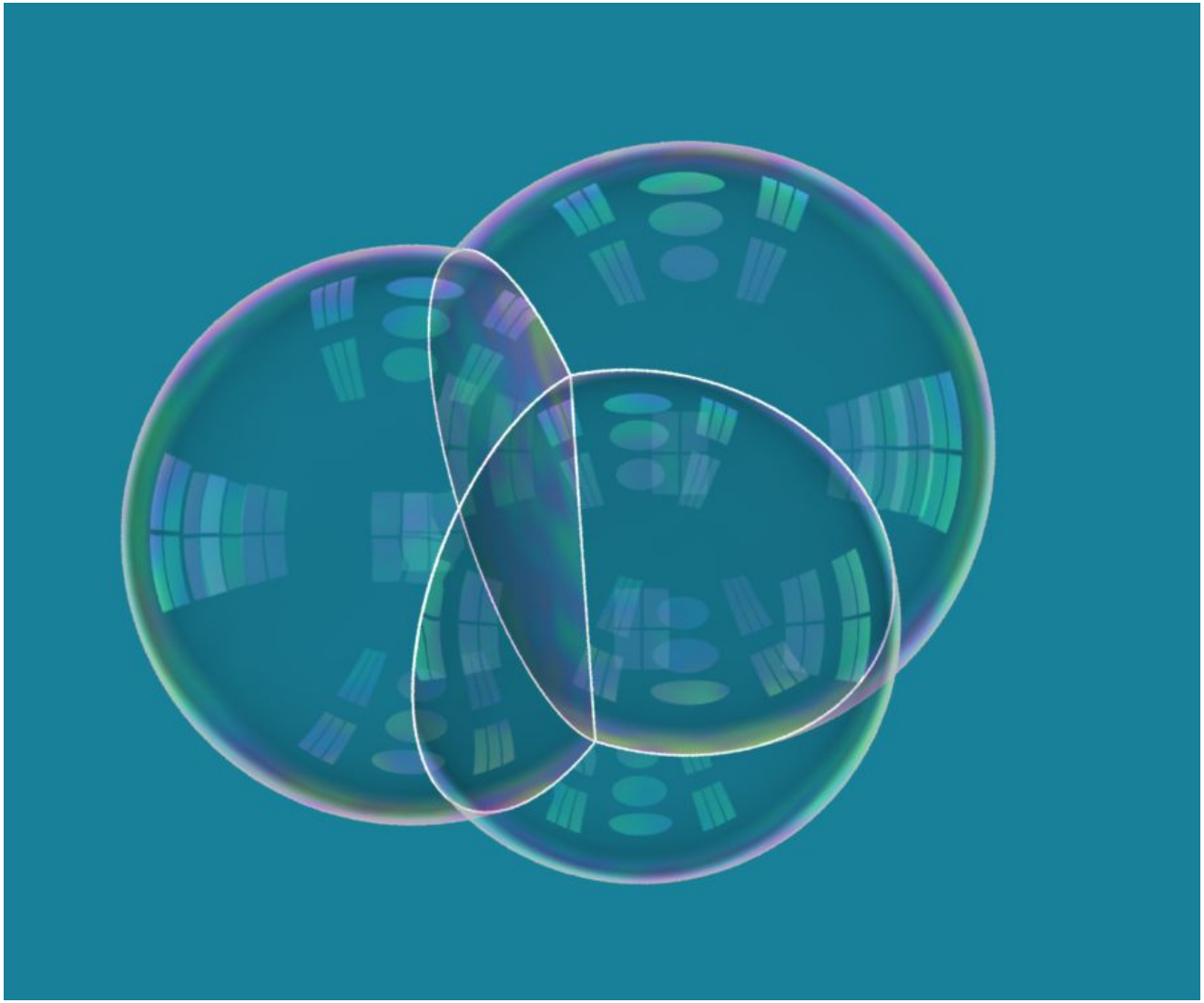


Plateau's first Rule

- Plateau borders
- triple junctions
- equal $120°$ angles

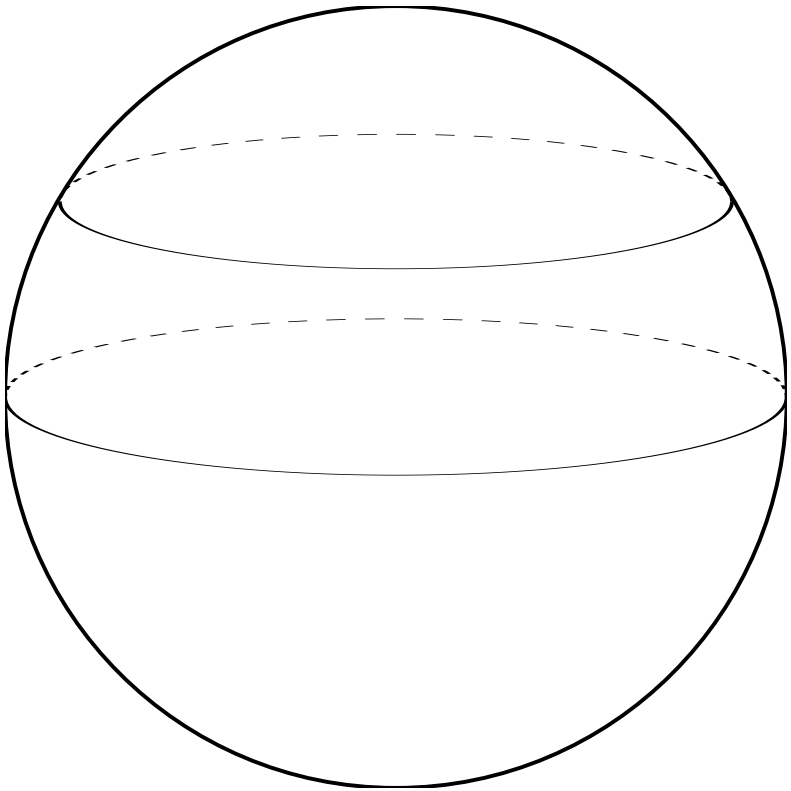


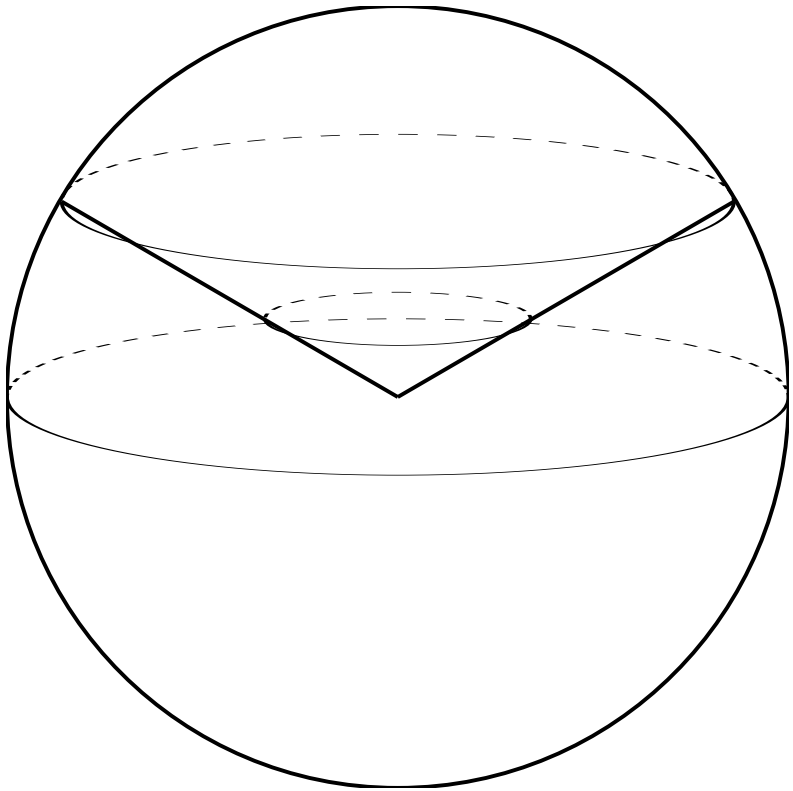
Point singularities



Cones

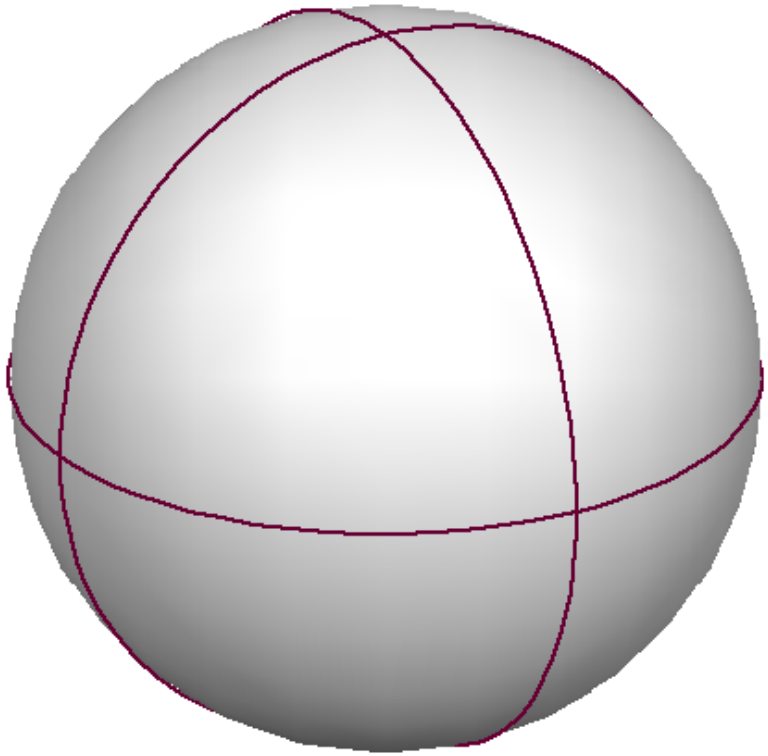
- Cone over a network in the sphere





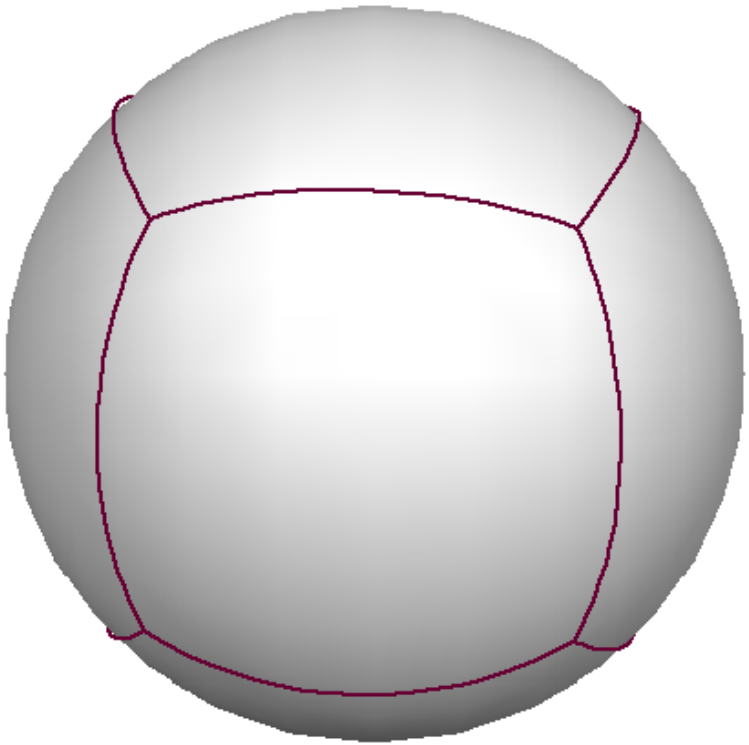
dual networks

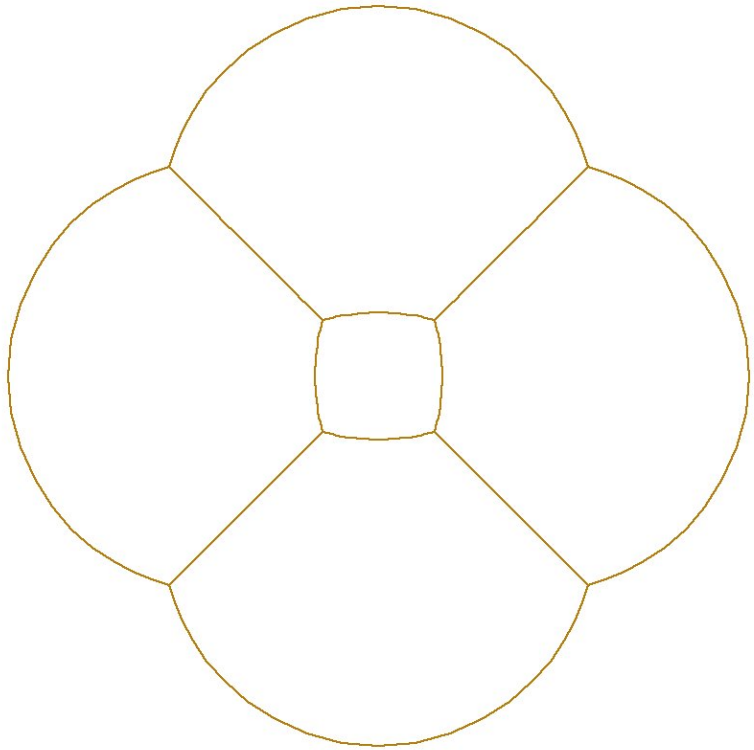
- dual net to cube
- spherical octahedron



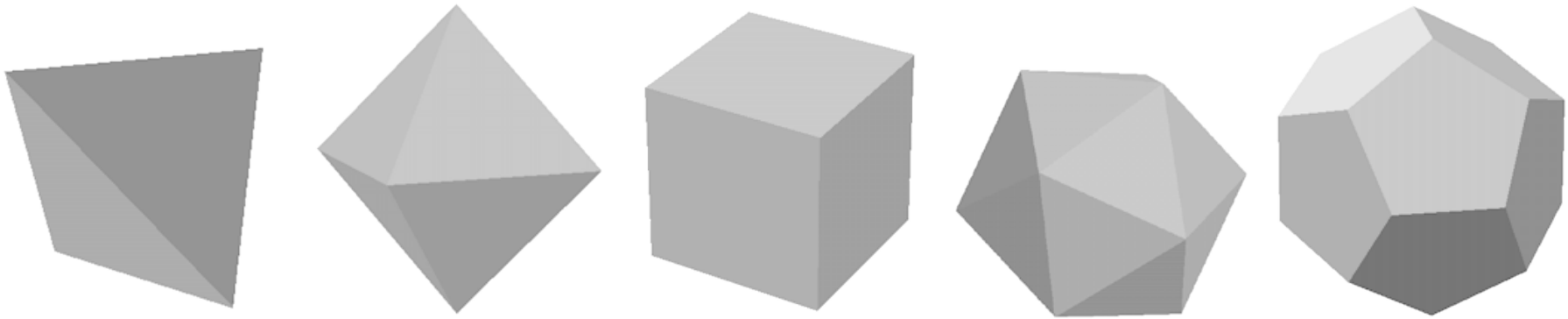
dual networks

- dual net to octahedron
- spherical cube
- stereographic projection gives 2D cluster





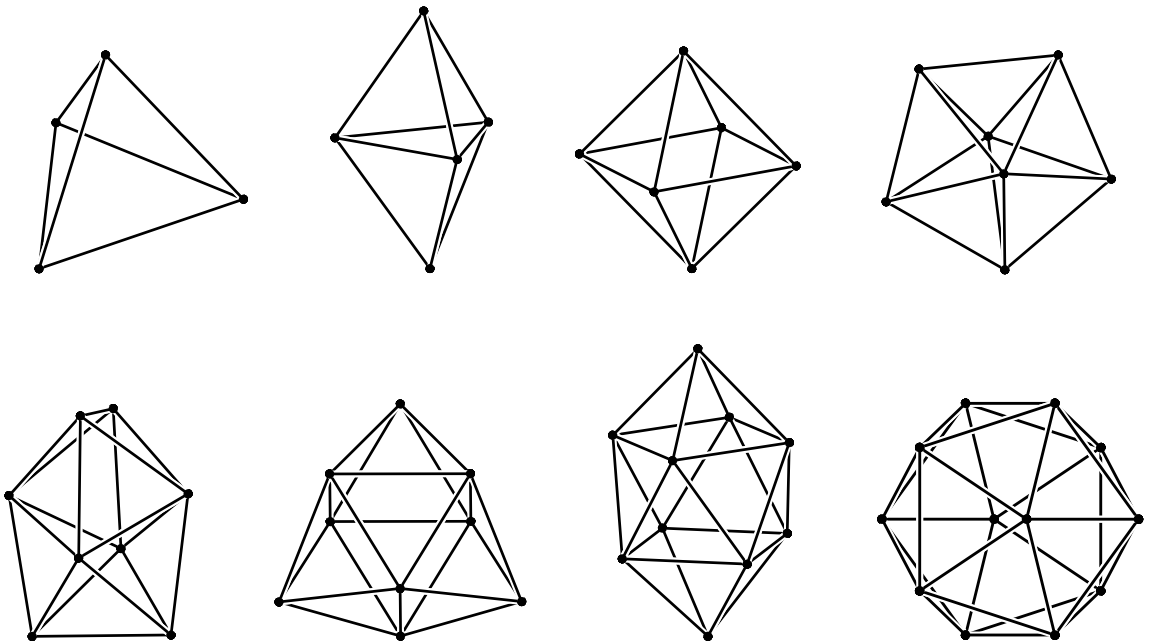
Platonic solids



- three have triangular faces
- tetrahedron, octahedron, icosahedron

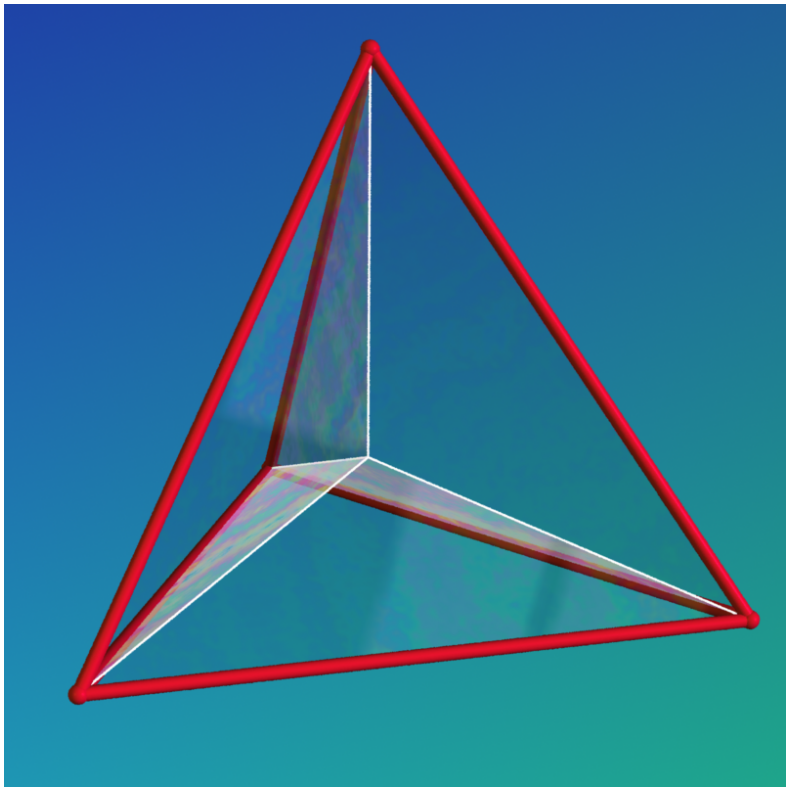
Convex deltahedra

- three Platonic solids
- two further bipyramids
- three further polyhedra



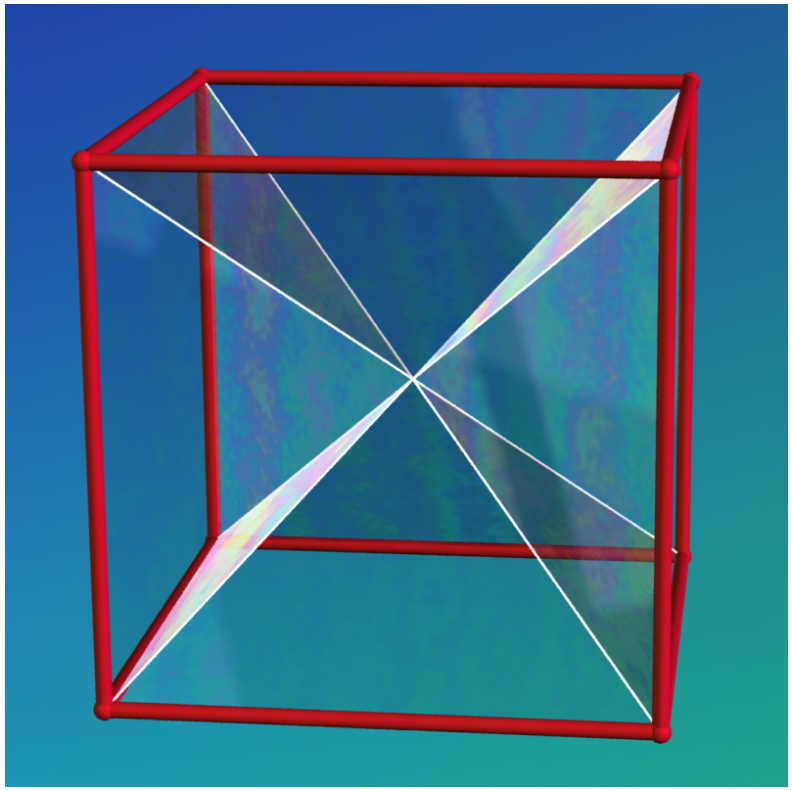
Cones in soapfilm

- Tetrahedron in soapy water



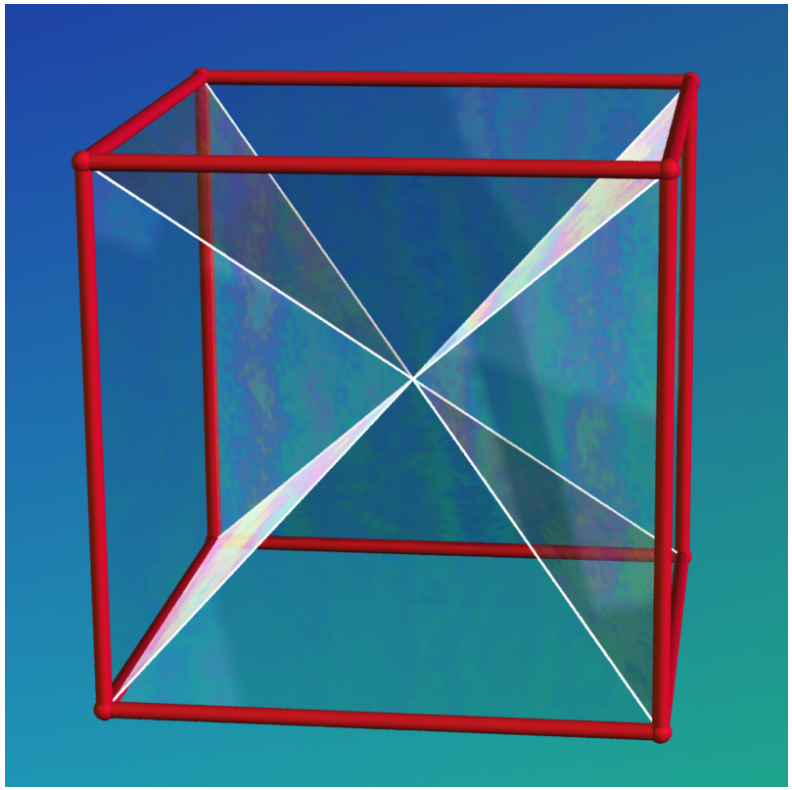
Cones in soapfilm

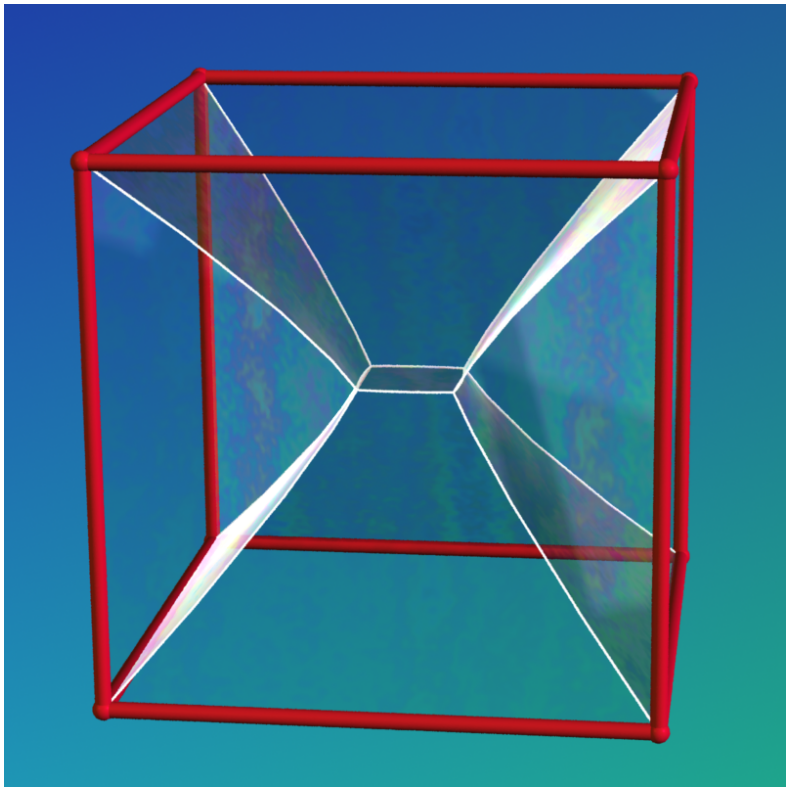
- Cube in soapy water?



Cones in soapfilm

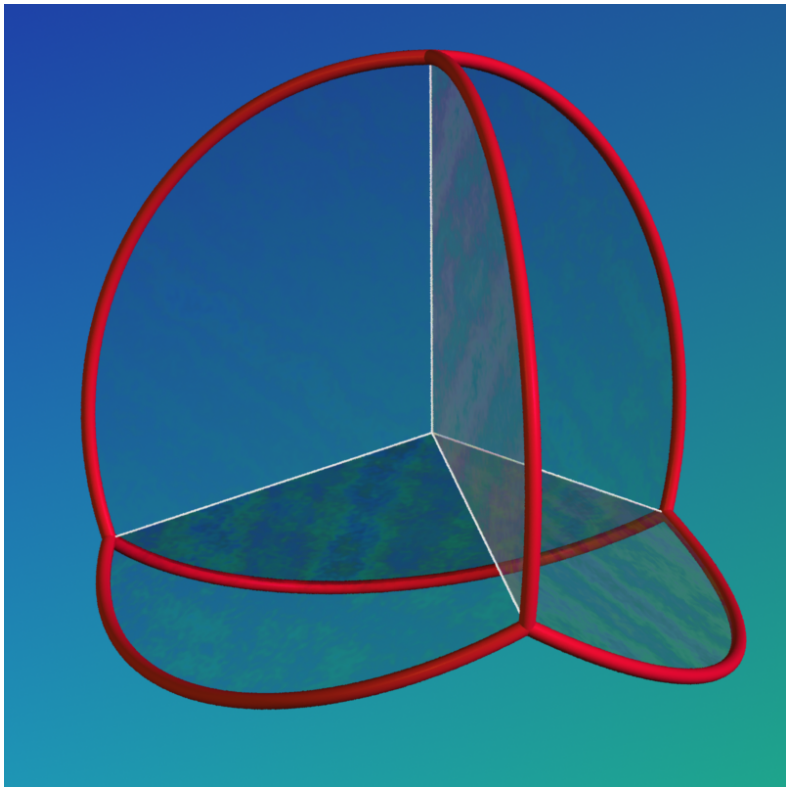
- Cube in soapy water





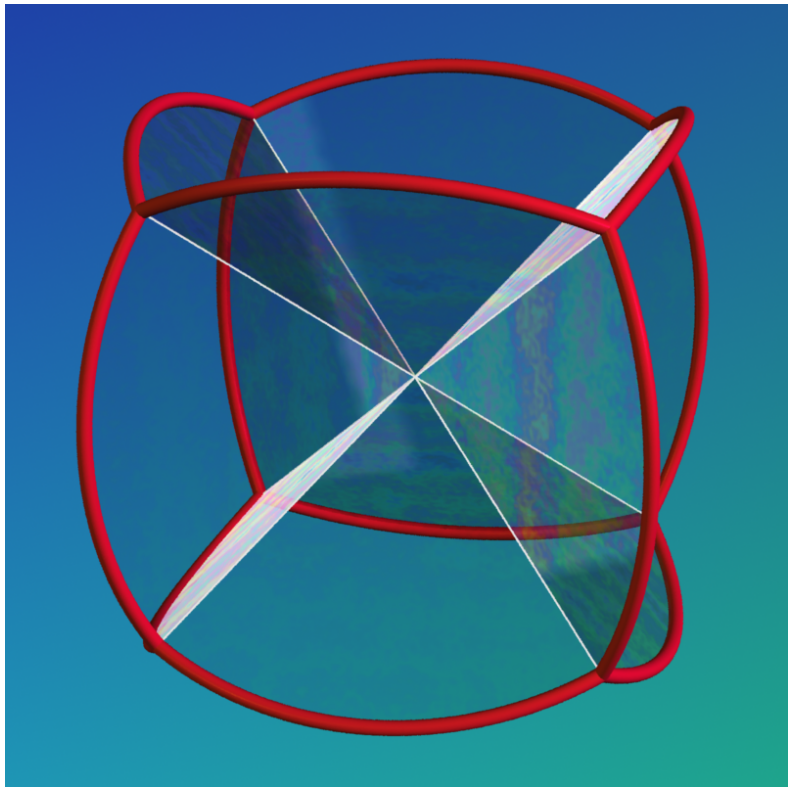
Cones in soapfilm

- Spherical tetrahedron in soapy water



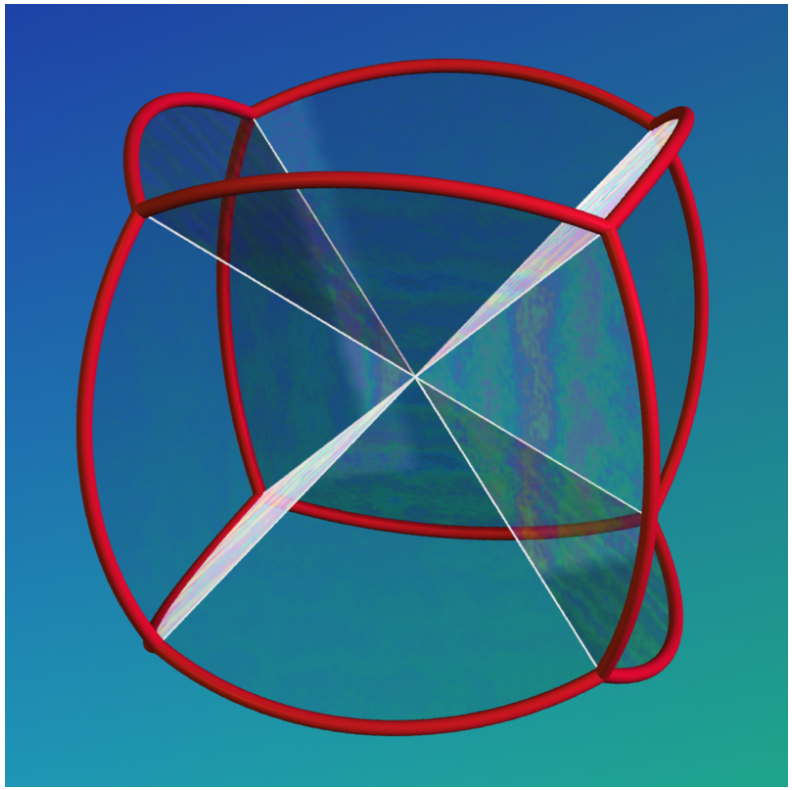
Cones in soapfilm

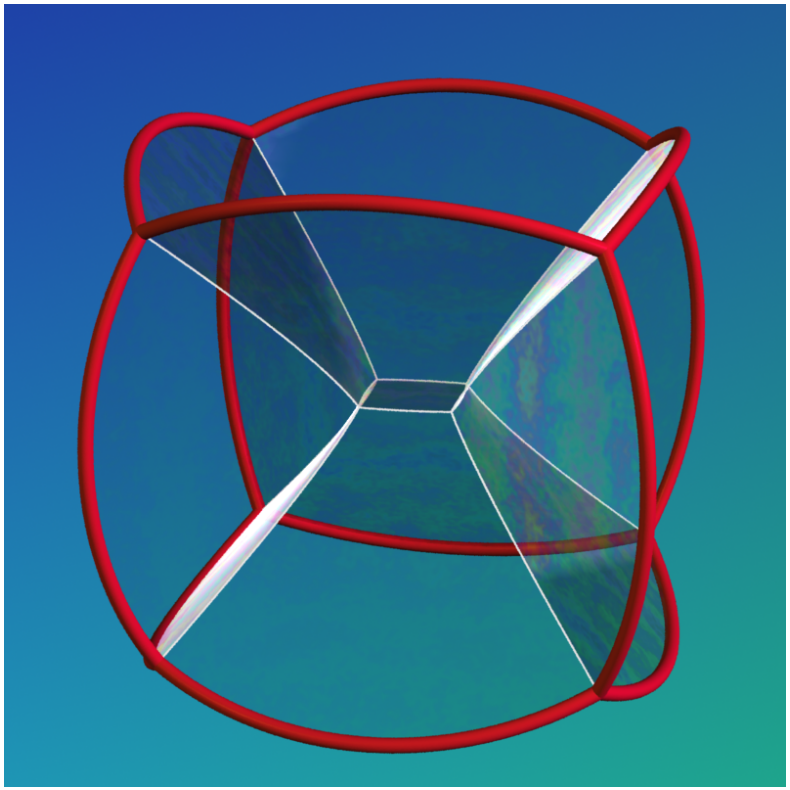
- Spherical cube in soapy water?



Cones in soapfilm

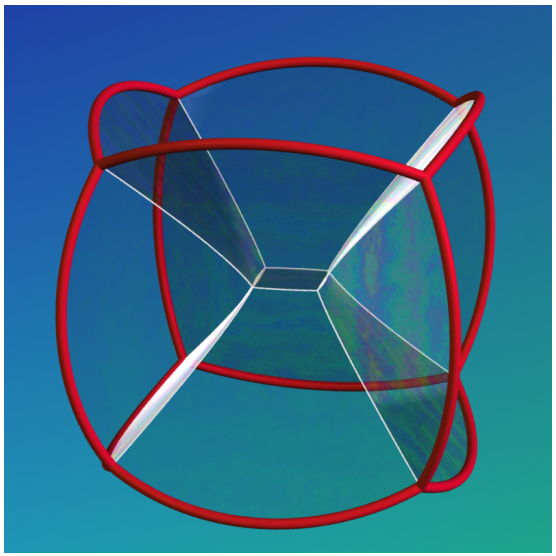
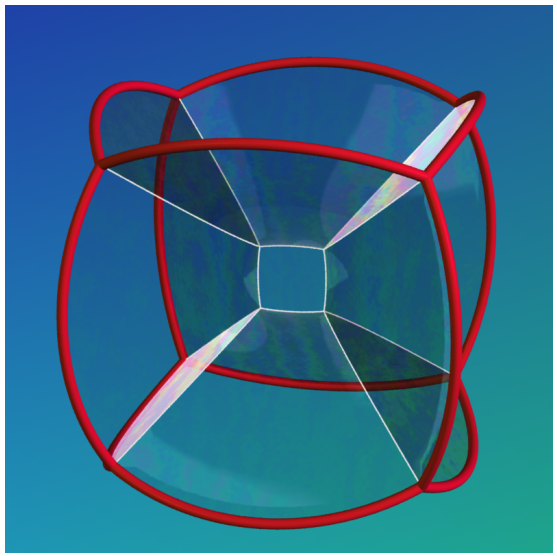
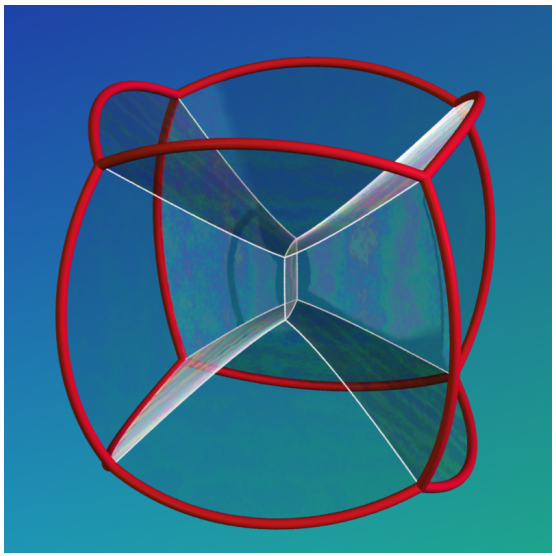
- Spherical cube in soapy water





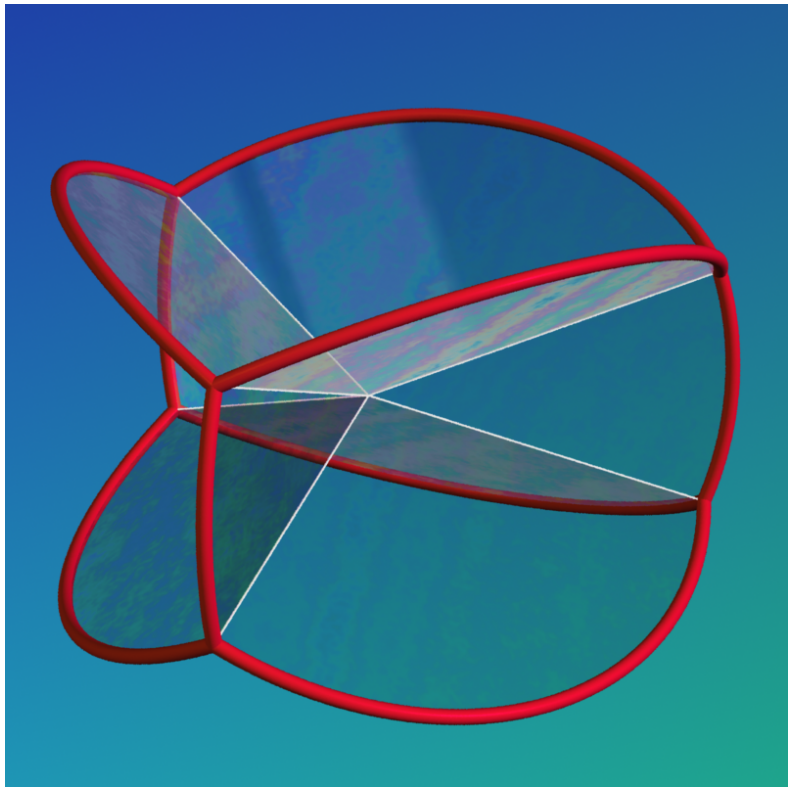
Cones in soapfilm

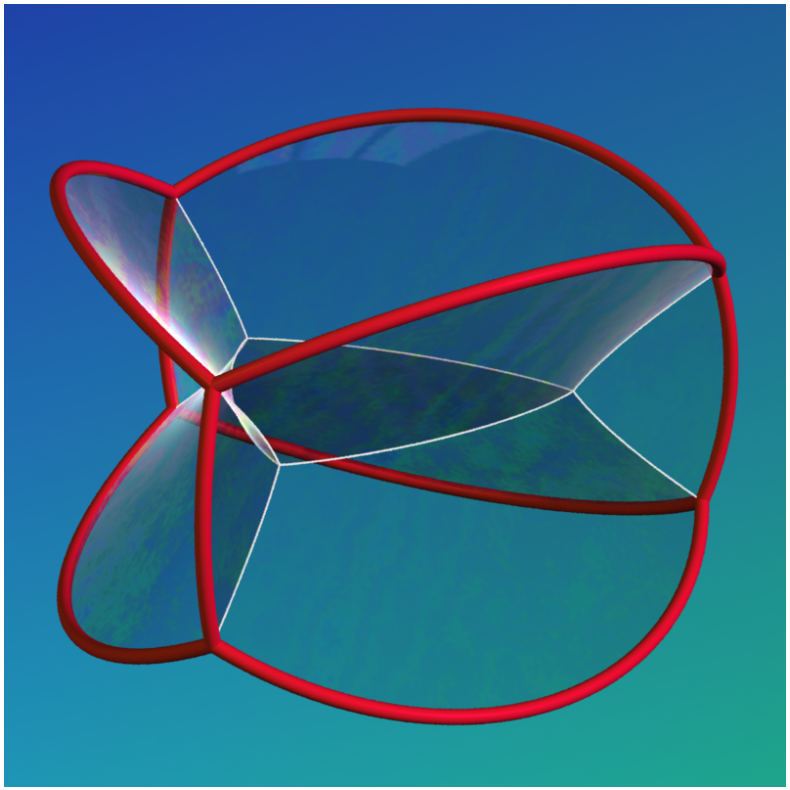
- Symmetry-breaking in 3 ways

Cones in soapfilm

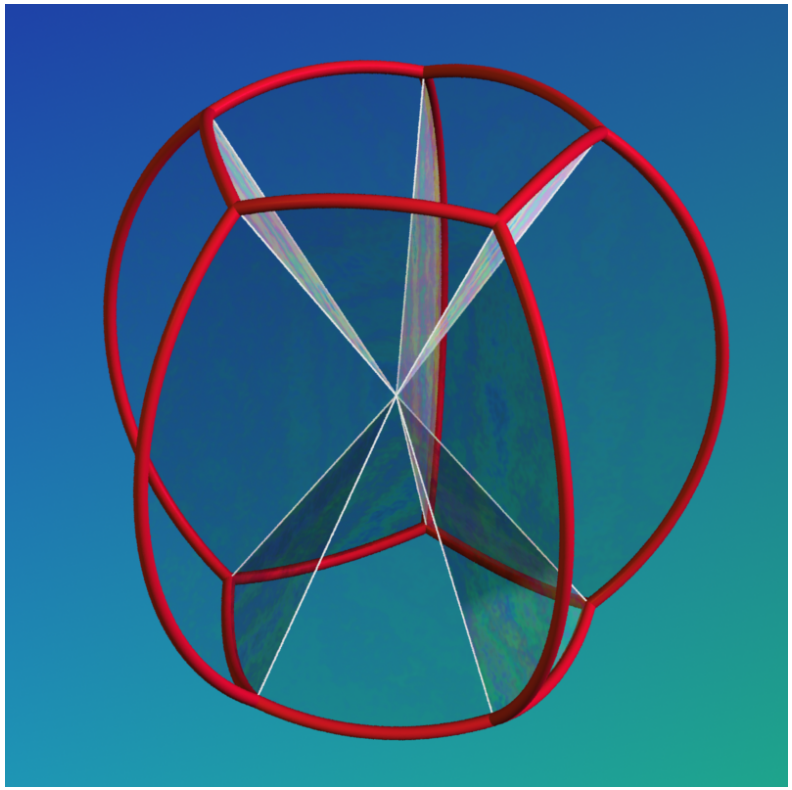
- Prism in soapy water

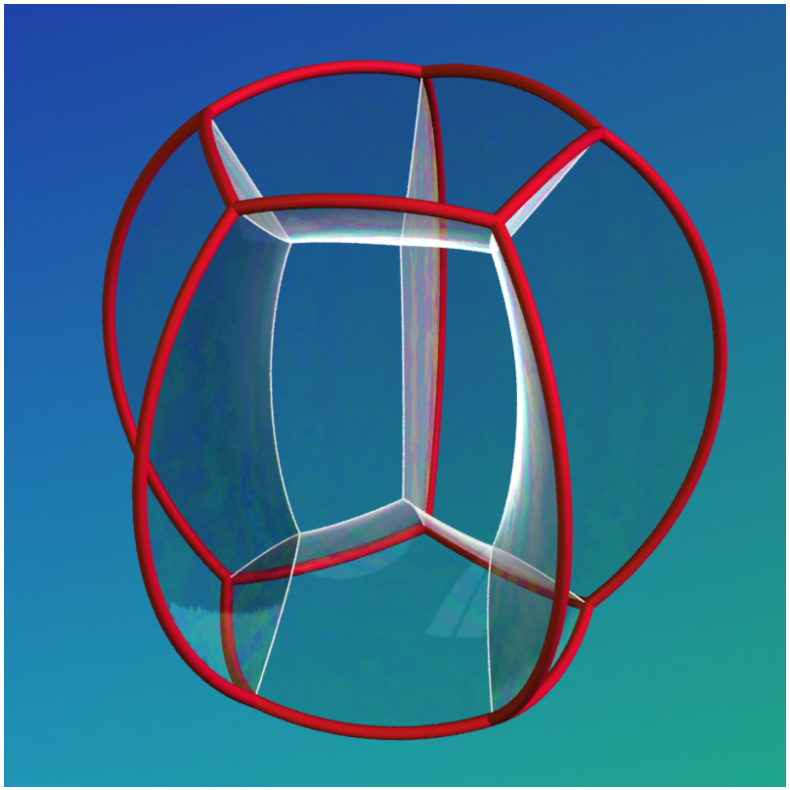




Cones in soapfilm

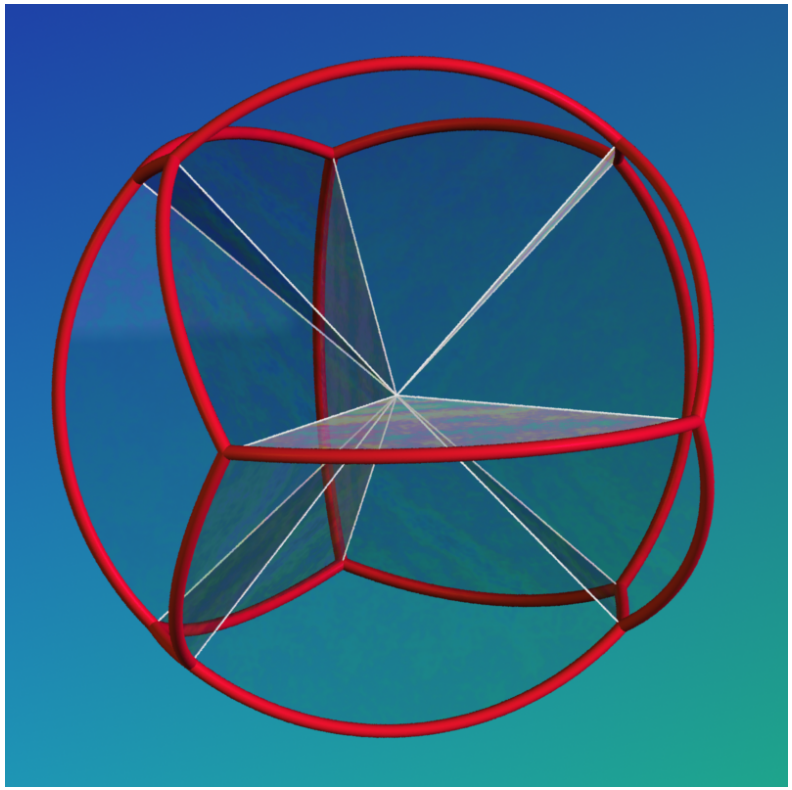
- Pentagonal prism in soapy water

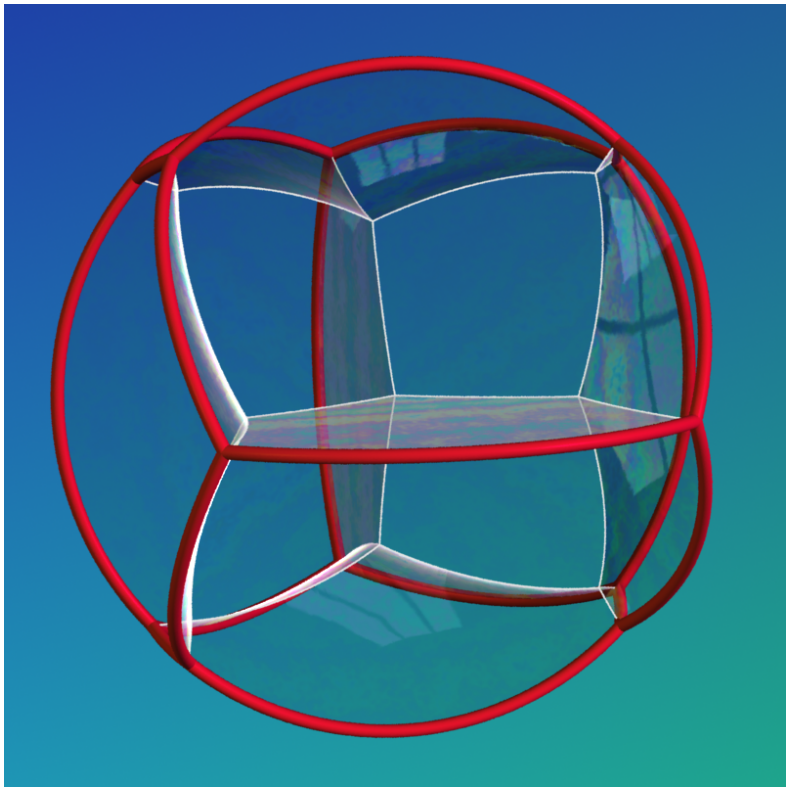




Cones in soapfilm

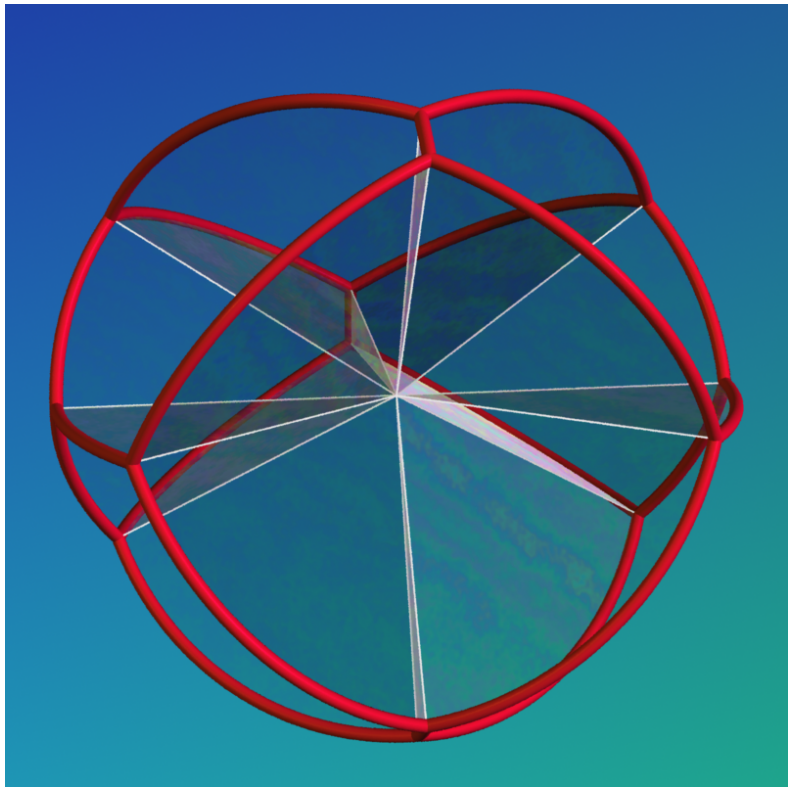
- 12-vertex net in soapy water

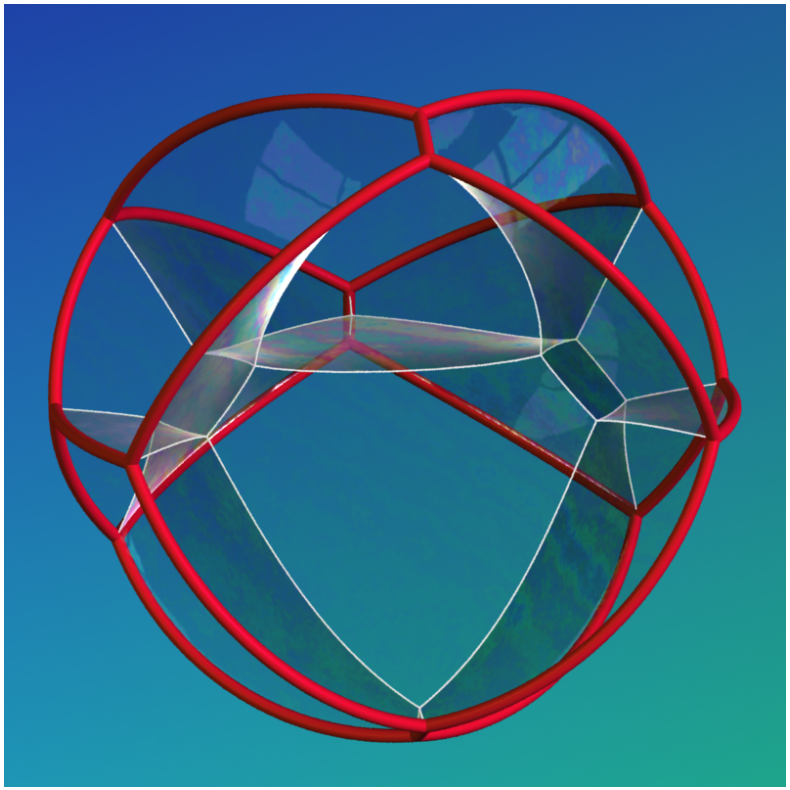




Cones in soapfilm

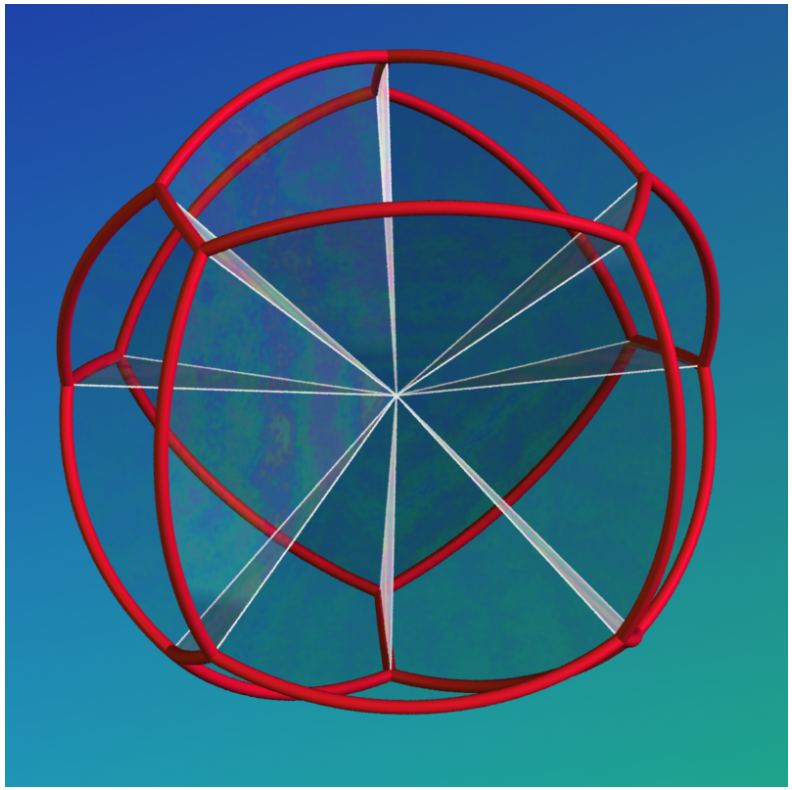
- 14-vertex net in soapy water

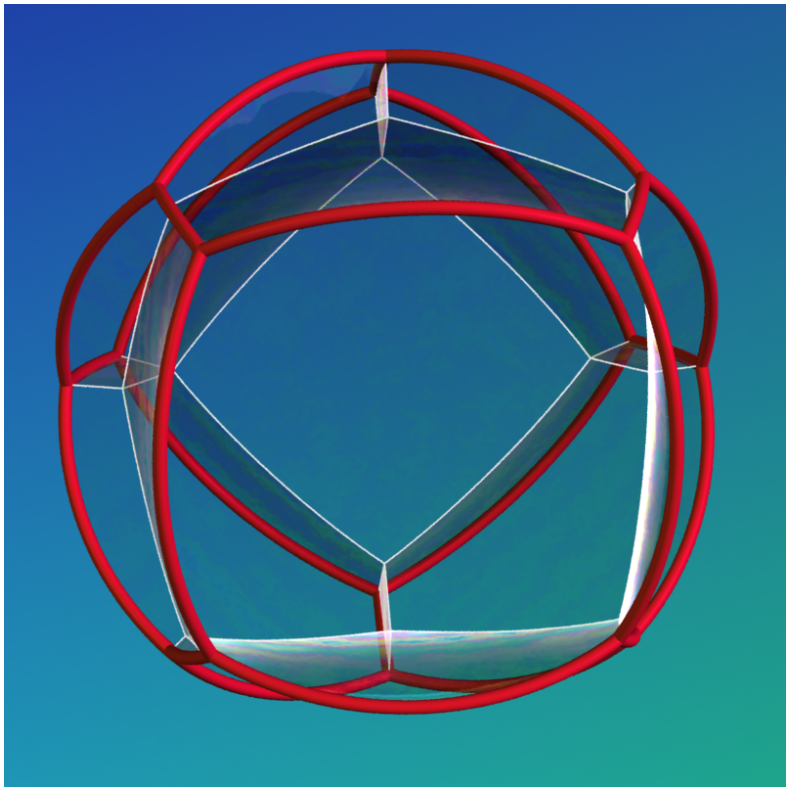




Cones in soapfilm

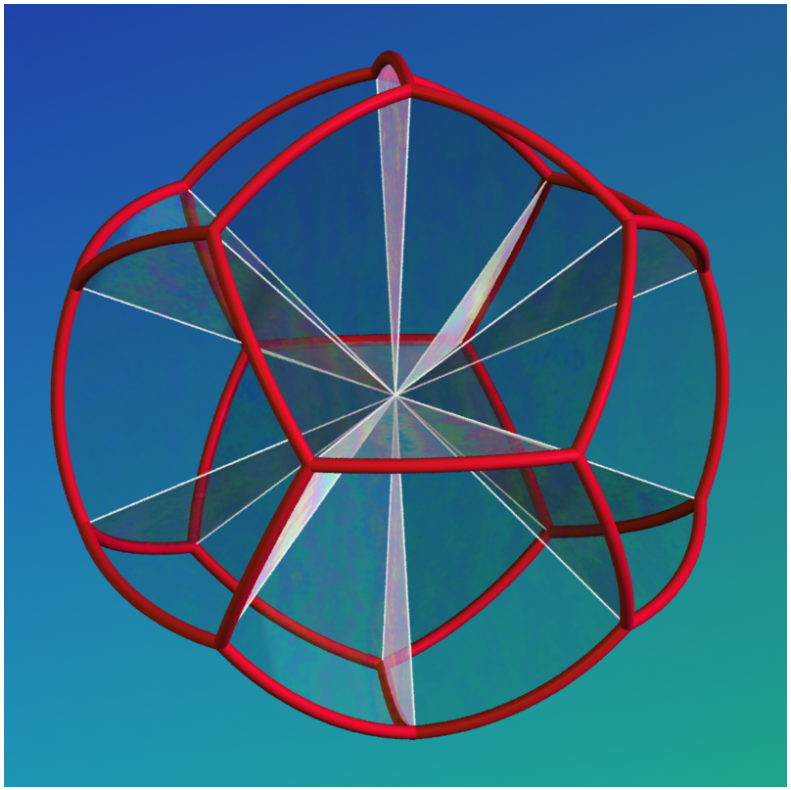
- 16-vertex net in soapy water

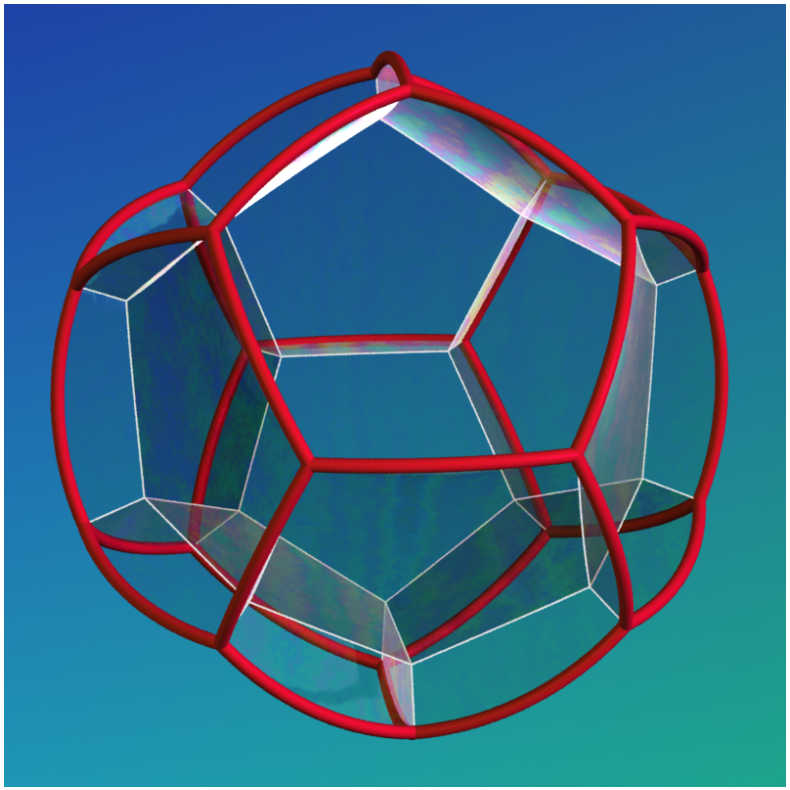




Cones in soapfilm

- Dodecahedron in soapy water





Higher dimensions

- Foams in the sphere
- from polyhedra in 4D