IUB International University Bremen

Mathematics Colloquium at IUB

Michael Lapidus

(UC Riverside)

will speak on

Can One Hear the Shape of a Fractal Drum?

Date:	**Monday, September 19, 2005**
Time:	**17:15**
Place:	**Lecture Hall Research II, IUB**

Abstract:

see next page.

Colloquium Tea at ca. 16:45 in the Tea Room of Research II, close to the lecture hall. Everybody is welcome!

M. Stoll

Abstract:

We discuss some aspect of the question (a la Mark Kac) “Can One Hear the Shape of a Fractal Drum?”. We will focus here on the case of a drum with fractal boundary and on the spectral asymptotics of the Dirichlet or Neumann Laplacian (or of more general and higher-order elliptic differential operators). In particular, we will pay attention to the one-dimensional case, that of fractal strings (or harps). In this situation, a suitable direct spectral problem (studied jointly with Carl Pomerance, and later on, in a brader context, with Christina He and Machiel van Frankenhuysen) is shown to be directly connected with the Riemann zeta function in the critical srip, while a corresponding inverse spectral problem is intimately connected with the Riemann hypothesis; namely, we show (jointly with Helmut Mayer) that one can hear whether a (noncritical) fractal string is Minkowski measurable if and only if the Riemann hypothesis is true.

If time allows, we will briefly explain how these intial results led naturally to a theory of complex fractal dimensions of fractal strings, which captures some of the essential features and the intrinsic oscillations of fractal and arithmetic geometries. It has since been developed in a series of papers and in the research monograph (joint with Machiel van Frankenhuysen) “Fractal Geometry and Number Theory: complex dimensions of fractal strings and zeros of zeta functions” (Birkhäuser, Boston, 2000, 280 pp.; second revised and enlarged edition in press, 2005, 440 pp.). The higher-dimensional theory is currently being developed with one of the speaker’s Ph.D. students, Erin Pearse. Finally, in a forthcoming book/essay, entitled “In Search of the Riemann Zeros: strings, fractal membranes, and noncommutative spacetimes” (approx. 510pp., May 2005), motivated in part by some aspects of the above theory and of modern theoretical physics (including standard string theory), a theory of quantized fractal strings (or ’fractal membranes’) is provided and the parallels uncovered between fractal, self-similar geometries and artihmetic geometries are further developed to provide a general framework in which to try to understand the Riemann hypothesis and its ramifications, in terms of a suitable flow on the moduli space of fractal membranes. A rigorous and noncommutative geometric version of part of this research program is currently being provided in joint work with Ryszard Nest, with whom very recently (i.e., over the last few months), we have begun developing a mathematical version of ‘complex homology/cohomology’ or ‘fractal cohomology theory’, as suggested in the abovementioned two research monographs.

Finally, we may mention very briefly another aspect of our research program, this time aimed at understanding in the long-term how fractal structures arise in nature, via the study of singularities of PDEs and numerical and graphical investigations. We refer, in particular, to earlier joint mathematical work with Michael Pang on the singularities of the gradient of the eigenfunctions of the Dirichlet Laplacian near a fractal boundary (such as the Koch snowflake curve), numerical and computer graphics-aided investigations with John W. Neuberger, Robert Renka and Cheryl Griffith, on the ’snowflake harmonics’ (i.e., the Dirichlet or Neumann eigenfunctions of the Koch snowflake drum or bell), as well as current joint work in progress with one of the speaker’s present Ph.D. students, Britta Daudert, on the localization of certain eigenfunctions and waves on Koch-type fractal drums.

Most (if not all) of the talk will be dedicated to a discussion of the earlier results regarding fractal drums and strings, as described in the first paragraph of this abstract. The talk should be self-contained and understandable to non-experts and graduate students.