IU$^{B}$ International University Bremen

# Mathematics Colloquium at IUB


## Claus Michael Ringel

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will speak on

*The Gabriel-Roiter Measure*

**Date:** **Monday, November 14, 2005**
**Time:** **17:15**
**Place:** **Lecture Hall Research II, IUB**


**Abstract:**

Let $k$ be a field and $A$ an associative $k$-algebra with 1. We consider the category of all finite-dimensional $A$-modules (these are finite-dimensional $k$-vector spaces on which $A$ acts). We may write any finite-dimensional $A$-module as a direct sum of indecomposable ones, and the theorem of Krull-Remak-Schmidt asserts that such a decomposition is unique up to isomorphism. Thus, one is interested in the set $\mathrm{ind}(A)$ of isomorphism classes of indecomposable $A$-modules. If this set is finite, one says that $A$ is representation-finite (usually $\mathrm{ind}(A)$ will be infinite). Typical examples of representation-finite algebras are the proper factor rings of the polynomial ring $k[X]$ in one variable, as well as the path algebras corresponding to the Dynkin diagrams $A_n$, $D_n$, $E_6$, $E_7$, $E_8$.

Around 1940, Brauer and Thrall formulated two conjectures concerning $\mathrm{ind}(A)$ for $A$ a finite-dimensional $k$-algebra. Here is the first one: Either $A$ is representation-finite, or else there do exist indecomposable $A$-modules of arbitrarily large dimension. The conjecture was proven by Roiter in 1968. The result has to be considered as the starting point of modern representation theory of finite-dimensional algebras.

One of the aims of the lecture is to explain this result by drawing the attention to an invariant of finite-dimensional modules which we call the Gabriel-Roiter measure. For representation-finite algebras this invariant was introduced by Gabriel already in 1973 in order to clarify the induction scheme used by Roiter in his proof of the first Brauer-Thrall conjecture. It turns out that it is of great importance when dealing with arbitrary finite-dimensional algebras. Using this invariant, we will discuss the global structure of such a module category, including questions concerning infinite-dimensional modules. Also, one obtains new insight into possible filtrations of indecomposable modules using indecomposable factors.


**Colloquium Tea** at ca. 16:45 in the Tea Room of Research II, close to the lecture hall. Everybody is welcome!

M. Stoll