

General Mathematics and Computational Science I

Exercise 2

September 19, 2006

1. Check whether each of the following relations is an equivalence relation, i.e. check whether it is reflexive, symmetric, and transitive. If a property holds, prove that it does. If a property does not hold, give a counter example.

(a) On \mathbb{Z} , define $x \sim y$ if and only if $x - y$ is divisible by 3.

Note: Any number $n \in \mathbb{Z}$ is divisible by 3 if there exists $k \in \mathbb{Z}$ such that $n = 3 \cdot k$.

(b) Let X be a nonempty set. Define, for any two subsets $A, B \subseteq X$, that $A \sim B$ if and only if $A \subseteq B$.

(c) On $\mathbb{Z} \times \mathbb{Z}$, define $(a, b) \sim (a', b')$ if and only if $ab' = ba'$.

2. Recall from class that we studied a binary operation $F: \mathbb{N} \times \mathbb{N} \rightarrow \mathbb{N}$ with the following properties:

(A1) $F(a, 1) = s(a)$ for all $a \in \mathbb{N}$,

(A2) $F(a, s(b)) = s(F(a, b))$ for all $a, b \in \mathbb{N}$,

where $s: \mathbb{N} \rightarrow \mathbb{N}$ is as in Peano's axioms.

Prove that if

$$F(a, c) = F(b, c)$$

for some $a, b, c \in \mathbb{N}$, then $a = b$.