WALKSHOP ON MATHEMATICAL PHYSICS 2025

1. Schedule

Thursday, June 19, 2025

09:00-09:40	David Mitrouskas (Institute of Science and Technology Austria)
09:40-10:20	Joachim Kerner (FernUniversität in Hagen)
10:20-10:50	*Coffee break*
10:50-11:30	Sascha Lill (Università degli Studi di Milano)
11:30-12:10	Paul Pfeiffer (University of Munich)
12:10-13:20	*Lunch break*
13:20-14:00	Daniel Fröhlich (University of Paderborn)
14:00-14:40	Javier Valentín Martín (University of Paderborn)
14:40-15:20	Theotime Girardot (Gran Sasso Science Institute, L'Aquila)
15:20-15:50	*Coffee break*
15:50-16:30	Siegfried Spruck (KIT, Karlsruhe)
16:30-17:10	Jacob Geisler (University of Braunschweig)

Friday, June 20, 2025

09:00-09:40	Wilhelm Kroschinsky (University of Bonn)
09:40-10:20	Merten Mlinarzik (University of Braunschweig)
10:20-10:50	*Coffee break*
10:50-11:30	Matthias Herdzik (University of Braunschweig)
11:30-12:10	Konstantin Merz (University of Braunschweig)
12:10-13:20	*Lunch break*
13:20-14:00	Badreddine Benhellal (TU Hamburg)
14:00-14:40	Patrizio Bifulco (FernUniversität in Hagen)
15:20-18:30	*Hike*
18:30-21:00	*Dinner*

2. Abstracts

2.1. **Thursday, June 19, 2025.**

Daniel Fröhlich, University of Paderborn

Title: *Ultraviolet Renormalization by Functional Integration*

Schedule: 13:20-14:00

Abstract: I present a simple method for performing ultraviolet renormalization of the spin-boson model. Using only basic stochastic techniques, I construct a functional integral representation of the model's Hamiltonian semigroup. This representation serves as a neat tool for carrying out the ultraviolet renormalization procedure.

Jacob Geisler, University of Braunschweig

Title: *Almost Commuting Self-Adjoint Operators and Iterated Commutator Estimates*

Schedule: 16:30-17:10

Abstract: Given two almost commuting self-adjoint operators, a new method for finding exactly commuting operators is presented. For this, a differential equation for self-adjoint Hilbert-Schmidt operators is introduced. Quantitative results are proven that the exactly commuting operators are close to the old ones in the Hilbert-Schmidt norm. The proof relies on a novel estimate in which the norm of the commutator is bounded from above by the norm of the iterated commutators times a constant. This inequality is proven in finite dimensions and lower bounds for the optimal constants are given.

Theotime Girardot, Gran Sasso Science Institute, L'Aquila

Title: *Derivation of the Chern–Simons–Schrödinger equation from the dynamics of an almost-bosonic-anyon gas*

Schedule: 14:40-15:20

Abstract: Anyons are 2D particles endowed with a statistical parameter interpolating between the symmetry of bosons and the antisymmetry of fermions. In this talk, anyons will be seen as bosons carrying an Aharonov-Bohm magnetic flux spread over a radius R. I will explain how the dynamics of an initial product state can be well approximated, in finite time, by a product state evolving under the effective Chern–Simons–Schrödinger equation for some specific regimes where R goes to zero as N becomes large.

Joachim Kerner, FernUniversität in Hagen

Title: *Atypical spectral and transport properties of non-locally finite crystals (and maybe more)*

Schedule: 09:40-10:20

Abstract: In the first part of the talk we discuss recent results on Schrödinger operators on periodic graphs which are non-standard in the sense that we allow vertices to have an infinite number of neighbours. It turns out that such non-locally finite graphs exhibit various phenomena which are absent in the locally finite setting: and this is true from a spectral as well as transport point of view. Using some explicit examples, we shall illustrate such new effects in more detail. Quite surprisingly, it turns out that one of the examples provides us with a negative answer to a question raised by Damanik et al. in a recent paper on ballistic transport (this part of talk is based on joint work with O. Post, M. Sabri, and M. Täufer). If time allows, we shall also quickly discuss spectral comparison results on discrete graphs. In recent years, various authors have derived such comparison results on Euclidean domains and quantum graphs. Our aim is to present a

generalization to the discrete setting. Along the way, we also establish a so-called local Weyl law which is of independent interest (the second part of the talk is based on joint work with P. Bifulco and C. Rose).

Sascha Lill, Università degli Studi di Milano

Title: *Friedrichs Diagrams-Bosonic and Fermionic*

Schedule: 10:50-11:30

Abstract: In Many-Body physics and QFT, one often encounters tedious computations of commutators involving creation and annihilation operators. A diagrammatic language introduced by Friedrichs in 1965 allows for cutting down these computations tremendously, while representing the occurring operators in a particularly convenient visual form. We revisit a formula for bosonic commutators in terms of Friedrichs diagrams and prove its fermionic analogue. The talk is based on joint work with Morris Brooks from the University of Zurich.

Javier Valentín Martín, University of Paderborn

Title: *Ultraviolet renormalization of Spin Boson Models with normal and 2-nilpotent interactions*

Schedule: 14:00-14:40

Abstract: Spin-Boson Models describe the interaction of a bosonic quantum field with a spin system. In this talk, we will study a generalized version of these models presenting ultraviolet divergences. For normal interactions, we construct a renormalized Hamiltonian via a dressing transformation, while for 2-nilpotent interactions, renormalization is achieved through the use of Interior Boundary Conditions (IBC). In both cases, the domain of the renormalized Hamiltonian is explicit, and the resulting operator can be obtained as the norm (or strong) resolvent limit of regular Hamiltonians. By combining both approaches, we obtain a more general renormalized model that incorporates both interaction types.

David Mitrouskas, Institute of Science and Technology Austria

Title: *The weakly coupled two-dimensional Fermi polaron*

Schedule: 09:00-09:40

Abstract: The Fermi polaron is a widely studied model in theoretical physics, describing a system of N fermions interacting with a single impurity particle via two-body point interactions. In this talk, we focus on the ground state energy of the Fermi polaron in two spatial dimensions. We show that in the weak coupling regime, the ground state energy is asymptotically given by the so-called polaron energy, as proposed by Frederic Chevy in the physics literature. This energy arises as the solution of a nonlinear equation involving the Green's function of the free Fermi gas and the binding energy associated with the two-body point interaction.

Paul Pfeiffer, University of Munich

Title: *TBA*

Schedule: 11:30-12:10

Abstract: TBA

Siegfried Spruck, KIT, Karlsruhe

Title: *Derivation of the Effective Dynamics for the Bose Polaron*

Schedule: 15:50-16:30

Abstract: We consider the dynamics of a dense quantum gas consisting of N bosons evolving in $\mathbb{R}^3$ in the presence of an impurity particle in the mean-field scaling with initially high density ρ and large volume Λ of the gas. In the initial state of the system almost all bosons are in the Bose-Einstein condensate, with a few excitations. For this system we derive from the microscopic dynamics in the limit of large densities and volumes the effective description by a quantum field theory modelled by the Bogoliubov-Fröhlich Hamiltonian which describes a quasi-particle, the Bose polaron.

2.2. **Friday, June 20, 2025.**

Badreddine Benhellal, TU Hamburg

Title: *Dirac operators with critical shell interaction in a finite box*

Schedule: 13:20-14:00

Abstract: We explore examples of Dirac operators on bounded domains exhibiting an interval of essential spectrum. In particular, we consider three-dimensional Dirac operators on Lipschitz domains with critical electrostatic and Lorentz scalar shell interactions supported on a compact smooth surface. Unlike typical bounded-domain settings where the spectrum is purely discrete, the criticality of these interactions can generate a non-trivial essential spectrum interval, whose position and length are explicitly controlled by the coupling constants and surface curvatures. Based on joint work with J. Behrndt (TU Graz), M. Holzmann (TU Graz), and K. Pankrashkin (Univ. Oldenburg).

Patrizio Bifulco, FernUniversität in Hagen

Title: *Spectral comparison results, now and then: An overview*

Schedule: 14:00-14:40

Abstract: We study Schrödinger operators on compact finite metric graphs subject to δ-coupling and standard boundary conditions often known as Kirchoff-Neumann vertex conditions. We compare the n-th eigenvalues of those self-adjoint realizations and derive an asymptotic result for the mean value of the eigenvalue deviations which represents a generalization to a recent result by Rudnick, Wigman and Yesha obtained for domains in $\mathbb{R}^2$ to the setting of metric graphs. We start this talk by introducing the basic notion of a metric graph and discuss some basic properties of heat kernels on those graphs afterwards. In this way, we are able to discuss a so-called local Weyl law which is relevant for the proof of the asymptotic main result. If time permits, we will also briefly discuss the case of δ'-coupling conditions and some possible generalizations on infinite metric graphs having finite total length or even combinatorial graphs. This talk is based on joint works with Joachim Kerner (Hagen), Delio Mugnolo (Hagen) and Christian Rose (Potsdam).

Matthias Herdzik, University of Braunschweig

Title: *On the Bogolubov-Hartree-Fock Energy of the Pauli-Fierz Hamiltonian*

Schedule: 10:50-11:30

Abstract: In this talk, a variational analysis of the Bogolubov-Hartree-Fock energy of the Pauli-Fierz Hamiltonian is presented, which builds upon the works of Bach, Breteaux, and Tzaneteas (2013) and of Bach and Hach (2022). The main result is a simplification of the BHF energy functional in the case of total momentum $\vec{p} = \vec{0}$, moreover it is shown that the energy-momentum relation in the BHF approximation fulfills $E_{\mathrm{BHF}}(\vec{0}) < E_{\mathrm{BHF}}(\vec{p})$ for any $\vec{p} \neq \vec{0}$.

Wilhelm Kroschinsky, University of Bonn

Title: *Time Evolution and Stability of Bose-Einstein Condensation in the Gross-Pitaevskii Regime*

Schedule: 09:00-09:40

Abstract: We revisit the time evolution and stability of Bose-Einstein condensates (BECs) in an initially trapped 3D bosonic system in the Gross-Pitaevskii regime. We provide a novel proof that the system continues to exhibit BEC after the trapping potential is switched off, with its effective dynamics governed by the solution of the time-dependent Gross-Pitaevskii equation. The main strategy is the control a renormalized number of excitations with respect to the Schrödinger dynamics $t \mapsto e^{-itH_N}$ instead of controlling the number of excitations with respect to a suitable excitation dynamics $t \mapsto e^{-B_t} e^{-itH_N}$. This approach considerably simplifies some previous results in the literature.

Konstantin Merz, University of Braunschweig

Title: *On the electron distribution of relativistic atoms and heat kernel bounds*

Schedule: 11:30-12:10

Abstract: The study of the electron distribution in atoms and molecules is paramount in quantum physics and chemistry. By the uncertainty principle, the innermost electrons move with velocities which are a substantial fraction of the speed of light. Hence, a relativistic description is mandatory. In this talk, we present new pointwise upper bounds for the sum of the squares of the eigenfunctions of the relativistic Chandrasekhar operator, in particular for each angular momentum channel separately. Our proof is concise and primarily relies on recently established heat kernel bounds for Hardy perturbations of subordinated Bessel heat kernels. This talk is based on joint works with Krzysztof Bogdan and Tomasz Jakubowski, and with Rupert Frank.

Merten Mlinarzik, University of Braunschweig

Title: *On the Bogolubov-Hartree-Fock Energy of the Pauli-Fierz Hamiltonian at zero total Momentum*

Schedule: 09:40-10:20

Abstract: The interaction of a free, non-relativistiv electron with the quantized radiation field at zero total momentum is described by the Pauli-Fierz Hamiltonian $H_0^{g,\Lambda}$ with coupling constant $g \geq 0$ and ultraviolet cutoff $\Lambda < \infty$. We study the Bogolubov-Hartree-Fock energy $E_{\mathrm{BHF}}^{g,\Lambda}$, which results from minimizing the expectation of $H_0^{g,\Lambda}$ over (pure) quasi-free states, for Λ tending to infinity.

Using a parametrization of these quasi-free states in terms of Bogolubov and Weil transformations, we derive an explicit functional $\mathcal{G}_{g,\Lambda}$ with $\inf \mathcal{G}_{g,\Lambda} \leq E_{\mathrm{BHF}}^{g,\Lambda}$. This functional fully captures the interacting parts of the model. We provide a variational analysis of the functional, including an analysis of the asymptotics of the infimum for $\Lambda \to \infty$. In particular, we show

$$cg\Lambda^{3/2} \leq \inf \mathcal{G}_{g,\Lambda} \leq Cg\Lambda^{3/2}.$$

Furthermore, we identify a family of highly explicit trail states which realize the above asymptotics.