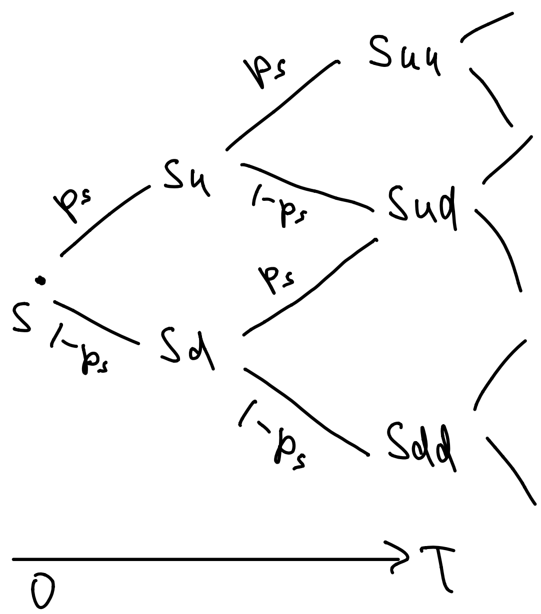
Session 8
Oct. 2, 2019

# 2.3 Binomial Tree Models

repeating the binary model with many steps yields a binomial tree

## Model for stock price development:




repeat n times

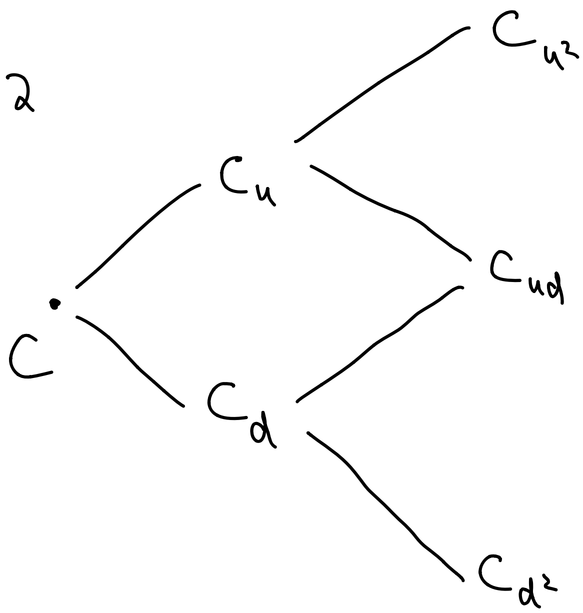
$\Rightarrow$ stock price $S_T^{j\,up} = S u^j d^{n-j}$ (n steps)

probability $P(j,n) = \underbrace{\binom{n}{j}}_{\frac{n!}{(n-j)!\,j!}} p_s^j (1-p_s)^{n-j}$

$\left(\text{remember } (a+b)^n = \sum_{j=0}^{n} \binom{n}{j} a^j b^{n-j}\right)$

length of period $= \frac{T}{n} = \Delta t$

## Option Price: $n=2$




for call options:

for $n=2$: start at last step (last column), at time of expiration

given: $C_{u^2} = \max(0, Su^2 - K)$ , $K$ = strike price

$C_{ud} = \max(0, Sud - K)$

$C_{d^2} = \max(0, Sd^2 - K)$

$C_u = e^{-r}\left(p\, C_{u^2} + (1-p)\, C_{ud}\right)$ where $p = p_u = \dfrac{e^r - d}{u-d}$

(as established for the binary model last time)

$C_d = e^{-r}\left(p\, C_{ud} + (1-p)\, C_{d^2}\right)$

next step: $C = e^{-r}\left(p\, C_u + (1-p)\, C_d\right)$

$$= e^{-2r}\left(p^2 C_{u^2} + 2p(1-p)\, C_{ud} + (1-p)^2 C_{d^2}\right)$$

in this case we get an explicit formula for the option price:

for n periods: $$C = e^{-nr} \sum_{j=0}^{n} \binom{n}{j} p^j (1-p)^{n-j} \max(0, S u^j d^{n-j} - X)$$

(note: in terms of the period interest rate $r_p$ we should use $r = r_p \frac{T}{n}$)

In the general case or with more complicated models (e.g. dividend payments or discontinuous interest compounding) there might not be closed-form formulas => better to implement bin. tree by "backward induction"

note: bin. tree model is very versatile (complicated models can be implemented in a simple way)