Moodle Exercise Set 6

Calculus and Linear Algebra II

Spring 2020

1. Suppose $(1,1)$ is a critical point of a continuously differentiable function f. Suppose that $\partial_{xx} f(1,1) = 4, \partial_{xy} f(1,1) = 1, \partial_{yy} f(1,1) = 2$. What can you say about f?
   A. f has a local maximum at $(1,1)$
   B. f has a local minimum at $(1,1)$
   C. f has a saddle point at $(1,1)$
2. Suppose $(0,-1)$ is a critical point of a continuously differentiable function f. Suppose that $\partial_{xx} f(0,-1) = 4, \partial_{xy} f(0,-1) = 3, \partial_{yy} f(0,-1) = 2$. What can you say about f?
   A. f has a local maximum at $(0,-1)$
   B. f has a local minimum at $(0,-1)$
   C. f has a saddle point at $(0,-1)$
3. What are the local maxima, local minima, and saddle points of $f(x,y) = 4 + x^3 + y^3 - 3xy$?
4. What are the local maxima, local minima, and saddle points of $f(x,y) = x^3 - 12xy + 8y^3$?
5. What are the local maxima, local minima, and saddle points of $f(x,y) = x^4 + y^4 - 4xy + 2$?
6. For functions of one variable it is impossible for a continuous function to have two local maxima and no local minima. However this is not the case for functions of two variables. Let $f(x,y) = -(x^2-1)^2 - (x^2y - x - 1)^2$. What are all the critical points of f? Which of the critical points are the local maxima?
7. Find three positive numbers whose sum is 100 and whose product is maximum. What is the product?
8. Find the points on the cone $z^2 = x^2 + y^2$ that are closest to the point $(4,2,0)$.
9. A cardboard box without a lid is to have a volume of 32000 cm^3. Find the dimensions that minimize the amount of cardboard used.
10. Use the method of Lagrange multipliers to find the extreme values of the function $f(x,y) = x^2 + y^2$ subject to the constraint $xy = 1$.
11. Use the method of Lagrange multipliers to find the extreme values of the function $f(x,y,z) = x^2 + y^2 + z^2$ subject to the constraint $x^4 + y^4 + z^4 = 1$.