Moodle Exercise Set 10

Calculus and Linear Algebra II

Spring 2020

1. Compute the inverse of the matrix
$$H = \begin{bmatrix} 1 & a & c \\ 0 & 1 & b \\ 0 & 0 & 1 \end{bmatrix}$$
where a, b, c are arbitrary real numbers.

2. Compute the inverse of the matrix
$$R_\theta = \begin{bmatrix} \cos(\theta) & 0 & \sin(\theta) \\ 0 & 1 & 0 \\ -\sin(\theta) & 0 & \cos(\theta) \end{bmatrix}$$
where θ is an arbitrary real number.

3. Use Cramer's rule to solve for y in
$$A\vec{x} = \begin{bmatrix} 2 & 6 & 2 \\ 1 & 4 & 2 \\ 5 & 9 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \vec{b}.$$

4. Use Cramer's rule to solve for y in
$$\begin{aligned} ax + by + cz &= 1 \\ dx + ey + fz &= 0 \\ gx + hy + iz &= 0. \end{aligned}$$
You can assume that the relevant 3×3 matrix has non-zero determinant D.

5. Use Cramer's rule to solve for x_1 in
$$\begin{aligned} 2x_1 + x_2 &= 1 \\ x_2 + 2x_2 + x_3 &= 0 \\ x_2 + 2x_3 &= 0. \end{aligned}$$

6. An 5×5 matrix has eigenvalues $\lambda_1, \ldots, \lambda_5$. If $\lambda_1 = 2 + i$, $\lambda_2 = \frac{i}{\sqrt{2}}$, $\lambda_3 = 2$ then what are the values of λ_4 and λ_5?

7. Let A be an $n \times n$ matrix that has eigenvalues $1, 3, 5, \ldots, 2n - 1$. Compute $\mathrm{tr}(A)$ and $\det(A)$.

8. Let A be a matrix such that
$$\det(A - \lambda I) = -\lambda^3(\lambda - 1)(2\lambda + 1)^2.$$
Compute $\mathrm{tr}(A)$ and $\det(A)$.

9. Let A be a 7×7 matrix such that
$$\det(A - \lambda I) = (\lambda - 2 + i)(\lambda - i)(\lambda - \sqrt{2})^2(\lambda - 1)q(\lambda)$$
where $q(\lambda)$ is some polynomial. Compute $\mathrm{tr}(A)$ $\det(A)$.

10. Consider the eigenvalues of the matrix

$$S = \begin{bmatrix} 1 & 2 & 2 & 3 & 4 & 1 \\ 2 & 4 & 3 & 5 & 5 & 7 \\ 2 & 3 & 9 & 1 & 1 & 0 \\ 3 & 5 & 1 & 0 & 1 & 2 \\ 4 & 5 & 1 & 1 & 3 & 1 \\ 1 & 7 & 0 & 2 & 1 & 8 \end{bmatrix}.$$

How many eigenvalues of the matrix are **not** real?

11. Consider the matrix

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}.$$

Compute the set of tuples $(\lambda, a_\lambda, g_\lambda)$ where λ is an eigenvalue, a_λ its algebraic multiplicity and g_λ is its geometric multiplicity.

12. Find the eigenvalues and the associated eigenvectors of the matrix

$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}.$$

13. Find the eigenvalues and the associated eigenvectors of the matrix

$$A = \begin{bmatrix} 7 & 0 & -3 \\ -9 & -2 & 3 \\ 18 & 0 & -8 \end{bmatrix}.$$

14. Consider the matrix

$$A = \begin{bmatrix} 1 & 8 \\ 0 & 1 \end{bmatrix}.$$

Compute the set of tuples $(\lambda, a_\lambda, g_\lambda)$ where λ is an eigenvalue, a_λ its algebraic multiplicity and g_λ is its geometric multiplicity.

15. Consider the matrix

$$A = \begin{bmatrix} 1 & 0 & 0 \\ -3 & 3 & 0 \\ 3 & 2 & 2 \end{bmatrix}.$$

Compute the set of tuples $(\lambda, a_\lambda, g_\lambda)$ where λ is an eigenvalue, a_λ its algebraic multiplicity and g_λ is its geometric multiplicity.