3.2 Decision Analysis

We consider decisions to be made where consequences/outcomes are uncertain, e.g.,

- how much of a product sells,
- whether or not to invest in equipment, securities, production facilities, ...

In practice, the following problem often arises:

- Prior probabilities are available for different scenarios, based on past experiences or intuition;
- We can invest in testing or experimentation to reduce uncertainties (= find better probabilities), e.g., test a product in a small market first, or get a more thorough analysis from experts/consultants.

Goal: maximize expected profit (or minimize expected costs, etc.)

Guiding example for this chapter: Goferbroke Oil Co. (Hillier, Lieberman: Ch. 15)

Setting:
- Company holds land where there might be oil or not (if not, land is "dry").
- Decision: Drill or sell?

Alternatives:	Payoff (in 1000$) in state: Oil	Dry
Drill (costs 100)	700 (= 800 − 100)	−100
Sell	90	90
prior probability	$\frac{1}{4}$	$\frac{3}{4}$

Given the prior probabilities, the expected payoffs are ($p = \frac{1}{4}$):

expected payoff if we drill

- $\mathbb{E}[\text{Drill}] = 700p - 100(1-p) = \frac{700}{4} - \frac{300}{4} = 100$
- $\mathbb{E}[\text{Sell}] = 90$

=> Drilling seems preferable here.

Note: For what probability p is it worth drilling?

Want $700p - 100(1-p) \geq 90 \implies 800p \geq 190 \implies p \geq \frac{190}{800} \approx 0.24$

This is very close to $\frac{1}{4}$, so maybe some experimentation is advisable.

For this example: We can do a seismic survey to find better probabilities:

- cost: 30 (k$)
- probabilities: $\mathbb{P}(\text{Favorable} \mid \text{Oil}) = 0.6$ (// $\mathbb{P}(F \mid \text{Oil})$), $\mathbb{P}(\text{Unfavorable} \mid \text{Oil}) = 0.4$ (// $\mathbb{P}(\text{Unf} \mid \text{Oil})$)
  $\mathbb{P}(\text{Favorable} \mid \text{Dry}) = 0.2$, $\mathbb{P}(\text{Unfavorable} \mid \text{Dry}) = 0.8$ } conditional probabilities

$\mathbb{P}(\text{Favorable} \mid \text{Dry})$: probability for favorable outcome if land is dry

Now we should find probabilities for oil <u>given</u> favorable/unfavorable outcome, i.e.,
$\mathbb{P}(\text{Oil} \mid F)$, $\mathbb{P}(\text{Oil} \mid \text{Unf})$.

Recall: $\underbrace{\mathbb{P}(A \cap B)}_{\text{prob. for } A \text{ and } B} = \underbrace{\mathbb{P}(A \mid B)}_{\text{prob. for } A \text{ given } B} \underbrace{\mathbb{P}(B)}_{\text{prob. for } B} = \mathbb{P}(B \cap A) = \mathbb{P}(B \mid A)\mathbb{P}(A)$

$$\Rightarrow \mathbb{P}(A \mid B) = \frac{\mathbb{P}(B \mid A)\mathbb{P}(A)}{\mathbb{P}(B)} \qquad \text{(Bayes' rule)}$$

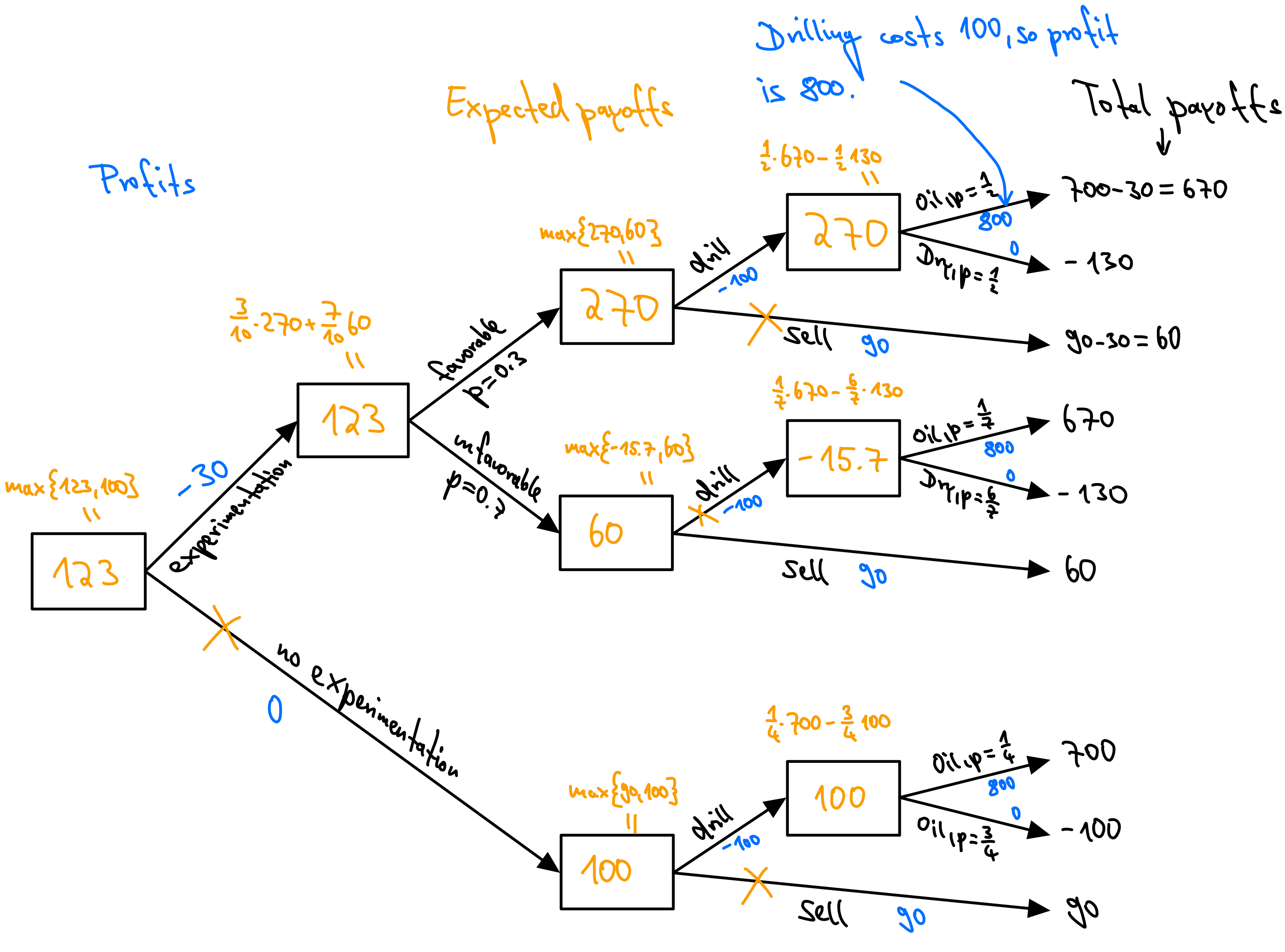
Let us first compute $\mathbb{P}(F)$:

$$\mathbb{P}(F) = \mathbb{P}(F \mid \text{Oil})\,\mathbb{P}(\text{Oil}) + \mathbb{P}(F \mid \text{Dry})\,\mathbb{P}(\text{Dry})$$
$$= 0.6\,\tfrac{1}{4} + 0.2\,\tfrac{3}{4}$$
$$= \tfrac{6}{40} + \tfrac{6}{40} = \tfrac{12}{40} = \tfrac{3}{10} = 0.3 \qquad \Rightarrow \mathbb{P}(\text{Unf}) = 0.7$$

$$\Rightarrow \mathbb{P}(\text{Oil} \mid F) = \frac{\mathbb{P}(F \mid \text{Oil}) \cdot \mathbb{P}(\text{Oil})}{\mathbb{P}(F)} = \frac{0.6 \cdot \frac{1}{4}}{0.3} = \frac{2}{4} = \frac{1}{2} = 0.5$$

$$\mathbb{P}(\text{Oil} \mid \text{Unf}) = \frac{0.4 \cdot \frac{1}{4}}{0.7} = \frac{1}{7}$$

This leads us to the following **decision tree**:



Result: • Do experimentation

• If favorable: drill

• If unfavorable: sell

• The overall expected profit is 123 000 \$.

Note:

• It is often useful to consider the expected value of experimentation (EVE)

= expected payoff with experimentation (not including cost of experimentation) − expected payoff without experimentation

$= (123+30) - 100 = 53$

Here, $53 > 30$ (cost of experimentation), so exp. should be done.

Generally: If EVE > cost of experimentation, then exp. should be done.

- If exp. would lead to perfect outcomes, we should consider the expected value of perfect information (EVPI)

$$= \text{expected payoff if state is perfectly known after exp.} - \text{expected payoff without exp.}$$

$$= \left(\frac{1}{4} \cdot 700 + \frac{3}{4} 90\right) - 100$$

$$= 142.5$$ ← if this were less than 30, then exp. would not be advisable, so sometimes EVPI can be used to exclude exp. (advantage: it is much easier to compute EVPI than EVE)