Week 11: Vectors

1. MULTI Single

A line is given by $\vec{r} = \lambda\vec{a} + \vec{b}$, with $\vec{a} = (1, -1, 4)^T$ and $\vec{b} = (4, 5, 6)^T$, while the equation of a plane is given by $-2x + 2y + z = 17$. What are the coordinates of the point P where the line and plane intersect?

(a) $P = (3, 3, 17)$
(b) $P = (-1, 4, 7)$
(c) The line and the plane intersect infinitely many times
(d) The line and the plane do not intersect

2. MULTI Single

What is the equation of the hyperplane, given by $\begin{bmatrix} t \\ x \\ y \\ z \end{bmatrix} = \vec{p}_0 + \alpha\vec{a} + \beta\vec{b} + \gamma\vec{c}$ with

$\vec{p}_0 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$, $\vec{a} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$, $\vec{b} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}$, $\vec{c} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$, $\alpha, \beta, \gamma \in \mathbb{R}$

(a) $t + x - y + z - 1 = 0$
(b) $-t - x - y + z + 1 = 0$
(c) $t + x - y - z - 1 = 0$
(d) $-t - x - y - z + 1 = 0$

3. MULTI Single

Find the cross product $\vec{u} \times \vec{v}$ of $\vec{u} = \langle 3, 2, -1\rangle$, $\vec{v} = \langle 1, 1, 0\rangle$

(a) $\langle -6, 4, 2\rangle$
(b) $\langle 1, -1, 1\rangle$
(c) $\langle -1, -1, 5\rangle$
(d) $\langle 6, 4, 2\rangle$

4. MULTI Single

Find the unit vector along the direction of the cross product $\vec{u}\times\vec{v}$ of $\vec{u} = \langle 7, -1, 3\rangle$, $\vec{v} = \langle 2, 0, -2\rangle$.

(a) $\frac{1}{408}\langle 2, 20, 2\rangle$
(b) $\frac{1}{\sqrt{108}}\langle -2, -10, 2\rangle$
(c) $\frac{1}{\sqrt{408}}\langle 2, 20, 2\rangle$
(d) $\frac{1}{108}\langle -2, -10, 2\rangle$

5. MULTI Single

Let $\epsilon_{ijk} = \begin{cases} 1 & \text{if } (i\,j\,k) = (1\,2\,3),\ (2\,3\,1),\ \text{or } (3\,1\,2) \\ -1 & \text{if } (i\,j\,k) = (1\,3\,2),\ (3\,2\,1),\ \text{or } (2\,1\,3) \\ 0 & \text{else} \end{cases}$

Consider $\vec{u} = \langle u_1, u_2, u_3 \rangle$ and $\vec{v} = \langle v_1, v_2, v_3 \rangle$. Which of the following is equivalent to the kth component of $\vec{u} \times \vec{v}$

(a) $[\vec{u} \times \vec{v}]_k = \sum_{i,j=1}^{3} \epsilon_{ijk}(u_i\, v_j - v_i\, u_j)$

(b) $[\vec{u} \times \vec{v}]_k = \sum_{i,j=1}^{3} \epsilon_{ijk} u_i\, v_j$

(c) $[\vec{u} \times \vec{v}]_k = \sum_{i,j=1}^{3} \epsilon_{ijk} v_i\, u_j$

(d) $[\vec{u} \times \vec{v}]_k = \sum_{i,j=1}^{3} \epsilon_{ijk}(u_i\, v_j + v_i\, u_j)$

6. MULTI Single

Find a basis for $\left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \in \mathbb{R}^3 \,\middle|\, 7x + 2y - 5z = 0 \right\} \subset \mathbb{R}^3$.

(a) $\begin{bmatrix} 5 \\ 0 \\ 7 \end{bmatrix}, \begin{bmatrix} 10 \\ 5 \\ 14 \end{bmatrix}$

(b) $\begin{bmatrix} 5 \\ 0 \\ -7 \end{bmatrix}, \begin{bmatrix} 0 \\ -5 \\ 2 \end{bmatrix}$

(c) $\begin{bmatrix} 5 \\ 0 \\ 7 \end{bmatrix}, \begin{bmatrix} 0 \\ 5 \\ 2 \end{bmatrix}$

(d) $\begin{bmatrix} 5 \\ 0 \\ -7 \end{bmatrix}, \begin{bmatrix} 0 \\ 5 \\ 2 \end{bmatrix}$

7. MULTI Single

Find a basis for $\left\{ \begin{bmatrix} 3a \\ -7a \\ 11a \end{bmatrix} \in \mathbb{R}^3 \,\middle|\, a \in \mathbb{R} \right\} \subset \mathbb{R}^3$.

(a) $\begin{bmatrix} 4 \\ 7 \\ 4 \end{bmatrix}$

(b) $\begin{bmatrix} -3 \\ -7 \\ 11 \end{bmatrix}$

(c) $\begin{bmatrix} 51 \\ -118 \\ 187 \end{bmatrix}$

(d) $\begin{bmatrix} 15 \\ -35 \\ 55 \end{bmatrix}$

8. MULTI Single

Which of the following is not a basis for the space of all cubic polynomials $P_3(\mathbb{R})$?

(a) $\mathfrak{B} = \{x^3, x^2, x, 1\}$
(b) $\mathfrak{B} = \{x^3 - x^2, x^2 - x, x - 1, 1\}$
(c) $\mathfrak{B} = \{x^3 - x^2, x^3 - x, x^2 - x, x^3 - 1\}$
(d) $\mathfrak{B} = \{x^3 + x^2 + x + 1, (x-6)^2, x - 10, 1\}$

9. MULTI Single

Does the set of all positive reals together with the following addition and multiplication by scalar $(\mathbb{R}_+, \tilde{+}, \tilde{\cdot})$ form a vector space over $\mathbb{R}$ (with the scalars $c \in \mathbb{R}$):

$$v_1 \tilde{+} v_2 \overset{def}{=} v_1 \cdot v_2; \quad c \tilde{\cdot} v_2 \overset{def}{=} c \cdot v_2$$

(a) $(\mathbb{R}_+, \tilde{+}, \tilde{\cdot})$ is a vector space over $\mathbb{R}$
(b) $(\mathbb{R}_+, \tilde{+}, \tilde{\cdot})$ is not a vector space over $\mathbb{R}$

10. MULTI Single

Is the set of all polynomials in one variable with real coefficients of degree 10 a vector space over $\mathbb{R}$ (let's denote this space by $\mathbb{R}[X]^{10}$)? (Addition and multiplication by scalar are defined as usual).

(a) $\mathbb{R}[X]^{10}$ is not a vector space over $\mathbb{R}$
(b) $\mathbb{R}[X]^{10}$ is a vector space over $\mathbb{R}$

Total of marks: 10