Week 12: Matrices

1. MULTI Single

Calculate the matrix product:

$$\begin{bmatrix} 1 & 2 & 9 \\ 3 & 4 & 5 \\ 6 & 7 & 8 \end{bmatrix} \cdot \begin{bmatrix} 1 & -1 & 1 \\ 1 & 1 & -1 \\ -1 & 1 & 1 \end{bmatrix} = ?$$

(a) $\begin{bmatrix} -6 & 10 & 8 \\ 2 & 5 & 4 \\ 5 & 9 & 7 \end{bmatrix}$

(b) $\begin{bmatrix} -6 & 9 & 8 \\ 2 & 6 & 4 \\ 5 & 9 & 8 \end{bmatrix}$

(c) $\begin{bmatrix} -6 & 9 & 8 \\ 2 & 5 & 4 \\ 5 & 9 & 8 \end{bmatrix}$

(d) $\begin{bmatrix} -6 & 10 & 8 \\ 2 & 6 & 4 \\ 5 & 9 & 7 \end{bmatrix}$

2. MULTI Single

Let

$$\mathcal{R} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

Which is the inverse of $\mathcal{R}$

(a) $\begin{bmatrix} -\cos\theta & -\sin\theta \\ \sin\theta & -\cos\theta \end{bmatrix}$

(b) $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

(c) $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

(d) $\begin{bmatrix} -\cos\theta & \sin\theta \\ -\sin\theta & -\cos\theta \end{bmatrix}$

3. MULTI Single

Let

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} B = \begin{bmatrix} 2 & 0 \\ 2 & 2 \end{bmatrix} C = \begin{bmatrix} 3 & 3 \\ 0 & 3 \end{bmatrix}$$

Calculate $A \cdot B \cdot C$

(a) $\begin{bmatrix} 6 & 6 \\ 6 & 6 \end{bmatrix}$

(b) $\begin{bmatrix} 12 & 18 \\ 12 & 18 \end{bmatrix}$

(c) $\begin{bmatrix} 24 & 24 \\ 2 & 6 \end{bmatrix}$

(d) $\begin{bmatrix} 6 & 6 \\ 12 & 18 \end{bmatrix}$

4. MULTI Single

Let

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} \; B = \begin{bmatrix} 99 & 0 \\ 99 & 99 \\ 99 & 0 \\ 99 & 99 \end{bmatrix} \; C = \begin{bmatrix} 3 \\ 0 \\ 3 \end{bmatrix}$$

Which of the following is a valid matrix multiplication?

(a) $A^T \cdot B^T \cdot C$
(b) $C^T \cdot B^T \cdot A^T$
(c) $B \cdot A^T \cdot C$
(d) $A \cdot B \cdot C$

5. MULTI Single

Which of the following is equivalent to $(A \cdot B \cdot C)^T$

(a) $A^T \cdot B^T \cdot C^T$
(b) $C^T \cdot B^T \cdot A^T$
(c) $B^T \cdot C^T \cdot A^T$
(d) $C^T \cdot B^T \cdot A^T$

6. MULTI Single

Let A be a (3×4) matrix, and B be a matrix such that $A^T \cdot B$ and $B \cdot A^T$ are both defined. What are the dimensions of B

(a) (4×3)
(b) (4×4)
(c) (3×4)
(d) (3×3)

7. MULTI Single

Solve the following system of linear equations:

$$\begin{aligned} x_1 + 3\,x_2 - 5\,x_3 &= 4 \\ x_1 + 4\,x_2 - 8\,x_3 &= 7 \\ -3\,x_1 - 7\,x_2 + 9\,x_3 &= -6 \end{aligned}$$

(a)

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -5 \\ 3 \\ 0 \end{bmatrix} + \lambda \begin{bmatrix} 4 \\ -3 \\ 1 \end{bmatrix}$$

(b)

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -10 \\ 6 \\ 0 \end{bmatrix} + \lambda \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}$$

(c)

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -10 \\ 6 \\ 0 \end{bmatrix} + \lambda \begin{bmatrix} 2 \\ -3 \\ -1 \end{bmatrix}$$

(d)

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -5 \\ 3 \\ 0 \end{bmatrix} + \lambda \begin{bmatrix} 4 \\ -3 \\ -1 \end{bmatrix}$$

8. MULTI Single

Find α such that following system of linear equations has no solutions:

$$\begin{aligned} x_1 + \alpha\, x_2 &= 1 \\ x_1 - x_2 + 3\, x_3 &= -1 \\ 2\, x_1 - 2\, x_2 + \alpha\, x_3 &= -2 \end{aligned}$$

(a) $\alpha = 6$
(b) $\alpha = 3$
(c) $\alpha = -1$
(d) $\alpha = 1$

9. MULTI Single

Which of the following is true for Homogeneous systems of Linear Equations?

(a) We can always find a solution $\vec{a}$ such that all its components a_i are positive
(b) If $\vec{a}$ is a solution, $\exists k \in \mathbb{R}$ such that $k\vec{a}$ is not a solution
(c) If $\vec{a}$ and $\vec{b}$ are both solutions, then $\vec{a} + \vec{b}$ is also a solution
(d) The system might not have a solution

10. MULTI Single

Let $\vec{a}$ and $\vec{b}$ be the solution to a system of linear equations $A\vec{x} = \vec{v}$. When is $\vec{a} + \vec{b}$ also a solution?

(a) When $\vec{v} = 0$
(b) Never
(c) Always
(d) When $\vec{v} \neq 0$

Total of marks: 10