Advanced Calculus and Methods of Mathematical Physics — Session 1

Prof. Sören Petrat, Jacobs University, Spring 2022 — Feb. 1, 2022

Organization:

- Prof. Sören Petrat (Mathematics)
  Office: room 112 in Research I
- class: Tue 11:15 – 12:30, Fri 8:15 - 9:30, in-person, Res. III lecture hall
  (note: software lab Fri 9:45 - 11:00 taught online by Jan Härter)
  Note: The 4 classes in the two weeks from Mar. 28 – Apr. 8 will be online.
- class website: news, syllabus, lecture notes, references, homework sheets
- moodle: – possibility for homework submission
  – possibly some homework solutions
- MS teams: selected online classes (Mar. 28 - Apr. 8)
- grade: – 100 % final exam
  – bonus: up to 10% from homework sheets (see website for details)
  (Note: bonus cannot change "fail" to "pass" grade.)
  – note: the total module grade is $\frac{2}{3}$ this class and $\frac{1}{3}$ the lab
- TA: Gandeeb Bhattarai
- homework sheets: – weekly, starting next week
  – hand in before class or upload to moodle
  – solutions discussed in weekly tutorial ← not mandatory, but highly recommended
- books: mostly Kantorovitz – Several Real Variables
  (see class schedule on website for more references)
- style of this class: – in between "Calculus and Linear Algebra I" and "Analysis I"
  – includes proofs, but avoids too much abstraction

<u>Topics:</u>

- Sequences and series of functions
- Differentiation in many variables → ~ $\frac{1}{3}$ of class
- Integration in many variables → ~ $\frac{1}{3}$ of class
- Fourier series/transform
- Complex analysis

1. Sequences and Series of Functions

1.1 Review of differentiation, integration, and Taylor's theorem

In this chapter, we consider functions $f: D \to \mathbb{R}$, with $D \subset \mathbb{R}$ (usually D is an interval or $D = \mathbb{R}$).

Let us recall a few important properties ($D \subset \mathbb{R}$ open):

- f is continuous at $\tilde{x} \in D$

  $:\Longleftrightarrow \underbrace{\forall}_{\text{"for all"}}$ sequences $(x_n)_{n\in\mathbb{N}}$ in D with $x_n \xrightarrow{n\to\infty} \tilde{x}$, we have $f(x_n) \xrightarrow{n\to\infty} f(\tilde{x})$ $\left(\lim_{n\to\infty} f(x_n) = f(\tilde{x})\right)$.

  We write this as $\lim_{x\to\tilde{x}} f(x) = f(\tilde{x})$.

  $\Longleftrightarrow f(\tilde{x}+h) = f(\tilde{x}) + R_{\tilde{x}}(h)$ with $\lim_{h\to 0} R_{\tilde{x}}(h) = 0$

  $\Longleftrightarrow \forall \varepsilon > 0 \ \underbrace{\exists}_{\text{"there exists"}} \delta > 0$ s.t. $\forall x \in D$: $|x - \tilde{x}| < \delta$ implies $|f(x) - f(\tilde{x})| < \varepsilon$

- f is differentiable at $\tilde{x} \in D$

  $\Longleftrightarrow f'(\tilde{x}) := \lim_{h\to 0} \frac{f(\tilde{x}+h) - f(\tilde{x})}{h}$ exists

  note: from this def. we see immediately that differentiability implies continuity

  $\Longleftrightarrow \exists m \in \mathbb{R}$ s.t. $f(\tilde{x}+h) = f(\tilde{x}) + mh + R(h)$ with $\lim_{h\to 0} \frac{R(h)}{h} = 0$

  (then $m = f'(\tilde{x})$ is the derivative of f at $\tilde{x} \in D$)

(Note: $f: D \to \mathbb{R}$ cont./diff.able $:\Longleftrightarrow f: D \to \mathbb{R}$ cont./diff.able $\forall \tilde{x} \in D$.)

Next time: Riemann integral and Taylor series

↳ need to talk about convergence of series of functions