Constructor University September 1, 2025
Fall 2025

Elements of Linear Algebra

Homework 1 (covering Weeks 1 and 2)

Due on September 17, 2025, before the tutorial! Please submit on moodle.

Problem 1 [6 points]
Prove the following identities for vectors $\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c} \in \mathbb{R}^3$.

1. The *"BAC–CAB-identity"*
$$\boldsymbol{a} \times (\boldsymbol{b} \times \boldsymbol{c}) = \boldsymbol{b}\,(\boldsymbol{a} \cdot \boldsymbol{c}) - \boldsymbol{c}\,(\boldsymbol{a} \cdot \boldsymbol{b})\,.$$
2. The *Jacobi identity* in three dimensions
$$\boldsymbol{a} \times (\boldsymbol{b} \times \boldsymbol{c}) + \boldsymbol{b} \times (\boldsymbol{c} \times \boldsymbol{a}) + \boldsymbol{c} \times (\boldsymbol{a} \times \boldsymbol{b}) = 0\,.$$

Problem 2 [6 points]
Prove the following identities for vectors $\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}, \boldsymbol{d} \in \mathbb{R}^3$.

1. The *Cauchy–Binet formula* in three dimensions
$$(\boldsymbol{a} \times \boldsymbol{b}) \cdot (\boldsymbol{c} \times \boldsymbol{d}) = (\boldsymbol{a} \cdot \boldsymbol{c})\,(\boldsymbol{b} \cdot \boldsymbol{d}) - (\boldsymbol{a} \cdot \boldsymbol{d})\,(\boldsymbol{b} \cdot \boldsymbol{c})\,.$$
Hint: Use the identity $\boldsymbol{u} \cdot (\boldsymbol{v} \times \boldsymbol{w}) = \boldsymbol{v} \cdot (\boldsymbol{w} \times \boldsymbol{u})$.
2. The identity
$$\|\boldsymbol{a} \times \boldsymbol{b}\|^2 = \|\boldsymbol{a}\|^2\,\|\boldsymbol{b}\|^2 - (\boldsymbol{a} \cdot \boldsymbol{b})^2\,.$$

Problem 3 [2 points]
Find an equation for the plane that contains the point $\boldsymbol{p} = (2, 4, 6)$ and the line
$$\boldsymbol{x} = \begin{pmatrix} 7 \\ 3 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} -3 \\ 4 \\ 2 \end{pmatrix}\,.$$

Problem 4 [6 points]
Find the distance between the point $\boldsymbol{p} = (1, 2, 3)$ and the line
$$\boldsymbol{x} = \begin{pmatrix} -1 \\ 1 \\ 6 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}\,.$$
Hint: Consider the square of the distance, and then set the derivative equal to zero to find the minimum.