

Elements of Linear Algebra

Homework 2 (covering Weeks 3 and 4)

Due on October 1, 2025, before the tutorial! Please submit on moodle.

Problem 1 [5 points]

Prove the following statement: Let $\mathbf{v}_1, \dots, \mathbf{v}_n$ be linearly independent. If a vector $\mathbf{w}$ can be written

$$\mathbf{w} = \sum_{k=1}^n \alpha_k \mathbf{v}_k,$$

the choice of the coefficients $\alpha_1, \dots, \alpha_n$ is unique.

Problem 2 [5 points]

Solve the following system of linear equations using the method taught in class.

$$\begin{aligned}x_1 + 3x_2 - 5x_3 &= 4 \\x_1 + 4x_2 - 8x_3 &= 7 \\-3x_1 - 7x_2 + 9x_3 &= -6\end{aligned}$$

Problem 3 [5 points]

Find conditions on α such that following system of linear equations has (a) exactly one solution, (b) no solutions, or (c) an infinite number of solutions; give all solutions where they exist.

$$\begin{aligned}x_1 + \alpha x_2 &= 1 \\x_1 - x_2 + 3x_3 &= -1 \\2x_1 - 2x_2 + \alpha x_3 &= -2\end{aligned}$$

Problem 4 [5 points]

Solve the following system of linear equations using the method taught in class.

$$\begin{aligned}x_1 + 3x_2 + x_3 + x_4 &= 2 \\2x_1 + 6x_2 - x_4 &= 1.\end{aligned}$$