Week 1: Basic Calculus Review

1. MULTI Single

   Find the (complex) roots of the polynomial

   $$p(x) = x^2 + 4\,x + 13$$

   (a) $x_1 = +2 + 3i$, $x_2 = +2 - 3i$
   (b) $x_1 = -3 + 2i$, $x_2 = -3 - 2i$
   (c) $x_1 = +3 - 2i$, $x_2 = +3 + 2i$
   (d) $x_1 = -2 - 3i$, $x_2 = -2 + 3i$

2. MULTI Single

   Find all the values of the parameter λ for which the equation

   $$2\,x^2 - \lambda\,x + \lambda = 0$$

   has no real solutions.

   (a) $\lambda \in \{0, 8\}$
   (b) $\lambda \in (0, 8)$
   (c) $\lambda \in (-8, 0)$
   (d) $\lambda \in (-\infty, 0) \cup (8, \infty)$

3. MULTI Multiple

   The number $5.21\overline{37}$ is:

   (a) an integer
   (b) a rational number
   (c) a natural number
   (d) a real number

4. MULTI Single

   Assuming that $z = a + bi$ is a complex number, compute real and imaginary part of $\dfrac{1}{z^2}$

   (a) $\text{Re}\left(\dfrac{1}{z^2}\right) = \dfrac{a^2 - b^2}{(a^2 - b^2)^2}, \text{Im}\left(\dfrac{1}{z^2}\right) = \dfrac{2ab}{(a^2 - b^2)^2}$
   (b) $\text{Re}\left(\dfrac{1}{z^2}\right) = \dfrac{a^2 + b^2}{(a^2 + b^2)^2}, \text{Im}\left(\dfrac{1}{z^2}\right) = \dfrac{2ab}{(a^2 + b^2)^2}$
   (c) $\text{Re}\left(\dfrac{1}{z^2}\right) = \dfrac{a^2 - b^2}{(a^2 + b^2)^2}, \text{Im}\left(\dfrac{1}{z^2}\right) = \dfrac{-2ab}{(a^2 + b^2)^2}$
   (d) $\text{Re}\left(\dfrac{1}{z^2}\right) = \dfrac{a^2 + b^2}{(a^2 + b^2)^2}, \text{Im}\left(\dfrac{1}{z^2}\right) = \dfrac{-2ab}{(a^2 + b^2)^2}$

5. MULTI Single

   Let $p(x)$ be a polynomial of degree n with **real** coefficients. Which of the following is true?

   (a) If $p(x)$ is odd, it can have no roots

(b) If z is a root, then its complex conjugate is z^* is also a root
(c) $p(x)$ has n distinct real roots
(d) $p(x)$ can have less than n complex roots

6. MULTI Single

Compute $\left|\frac{1+i}{2-i}\right|$.

(a) $\left|\frac{1+i}{2-i}\right| = \frac{2}{3}$
(b) $\left|\frac{1+i}{2-i}\right| = \frac{2}{5}$
(c) $\left|\frac{1+i}{2-i}\right| = \sqrt{\frac{2}{5}}$
(d) $\left|\frac{1+i}{2-i}\right| = \sqrt{\frac{2}{3}}$

7. MULTI Single

Which of the following does not describe the rational numbers $\mathbb{Q}$?

(a) $\mathbb{Q} = \left\{\frac{n}{m} \mid n \in \mathbb{Z} \text{ and } m \in \mathbb{N}\right\}$
(b) $\mathbb{Q} = \left\{\frac{n}{m} \mid n, m \in \mathbb{N}\right\} \cup \left\{\frac{-n}{m} \mid n, m \in \mathbb{N}\right\} \cup \{0\}$
(c) $\mathbb{Q} = \left\{\frac{n}{m} \mid n, m \in \mathbb{Z} \text{ and } m \neq 0\right\}$
(d) $\mathbb{Q} = \left\{\frac{n}{m} \mid n, m \in \mathbb{N}\right\}$

8. MULTI Single

Find the inverse f^{-1} of the function $f : [0, \infty) \to [1, \infty), x \mapsto x^2 + 1$.

(a) $f^{-1}(y) = \frac{1}{y^2 + 1}$
(b) $f^{-1}(y) = y^{-2} + 1$
(c) $f^{-1}(y) = \sqrt{y - 1}$
(d) $f^{-1}(y) = y^{-2} - 1$

9. MULTI Single

Let $g(x) = x^2 + 1$. Determine the domain and range of $g(x)$.

(a) $Domain(g) = [0, \infty)$, $Range(g) = (-\infty, \infty)$
(b) $Domain(g) = (-\infty, \infty)$, $Range(g) = [0, \infty)$
(c) $Domain(g) = (-\infty, \infty)$, $Range(g) = [1, \infty)$
(d) $Domain(g) = [0, \infty)$ $Range(g) = (-\infty, \infty)$

10. MULTI Single

Let $f(x) = 2^{-9x+3}$. Determine the domain and range of $f(x)$ and its inverse $f^{-1}(x)$.

(a) $Dom(f) = (-\infty, \infty)$, $Ran(f) = (0, \infty)$,
$Dom(f^{-1}) = (0, \infty)$, $Ran(f^{-1}) = (-\infty, \infty)$

(b) $Dom(f) = [0, \infty)$, $Ran(f) = [0, \infty)$,
$Dom(f^{-1}) = [0, \infty)$, $Ran(f^{-1}) = [0, \infty)$

(c) $Dom(f) = (0, \infty)$, $Range(f) = (-\infty, \infty)$,
$Dom(f^{-1}) = (-\infty, \infty)$, $Ran(f^{-1}) = (0, \infty)$

(d) $Dom(f) = (-\infty, \infty)$, $Ran(f) = [0, \infty)$,
$Dom(f^{-1}) = [0, \infty)$, $Ran(f^{-1}) = (-\infty, \infty)$

Total of marks: 10