

Week 5: Kernel, range, rank-nullity theorem, matrix inverse

- 1.
- ☒
- MULTI
- ☐
- Single

Find the inverse of $A = \begin{bmatrix} 0 & \frac{1}{2} & -\frac{1}{2} \\ 1 & 0 & 1 \\ 2 & \frac{1}{2} & 1 \end{bmatrix}$.

- (a) $A^{-1} = \begin{bmatrix} -2 & -3 & 2 \\ 4 & 4 & 2 \\ 2 & 4 & -2 \end{bmatrix}$
- (b) $A^{-1} = \begin{bmatrix} -2 & -3 & 2 \\ 4 & 4 & 2 \\ 2 & -4 & -2 \end{bmatrix}$
- (c) the inverse does not exist
- (d) $A^{-1} = \begin{bmatrix} -2 & -3 & 2 \\ 4 & 4 & -2 \\ 2 & 4 & -2 \end{bmatrix}$

- 2.
- ☒
- MULTI
- ☐
- Single

Find the inverse of $A = \begin{bmatrix} 3.5 & -1 & 0.5 \\ 10 & -3 & 2 \\ 2.5 & -1 & 1.5 \end{bmatrix}$.

- (a) $A^{-1} = \begin{bmatrix} 0 & 1 & 1 \\ 2 & -1 & -2 \\ -1 & 1 & -2 \end{bmatrix}$
- (b) $A^{-1} = \begin{bmatrix} 0 & 1 & 1 \\ 2 & -1 & -2 \\ 1 & 1 & 2 \end{bmatrix}$
- (c) the inverse does not exist
- (d) $A^{-1} = \begin{bmatrix} 0 & 1 & 1 \\ 2 & -1 & -2 \\ 1 & 1 & -2 \end{bmatrix}$

- 3.
- ☒
- MULTI
- ☐
- Single

Find the inverse of $A = \begin{bmatrix} 1 & -1 & 1 \\ -1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$.

- (a) $A^{-1} = \begin{bmatrix} 0.5 & -0.5 & 0.5 \\ -0.5 & 0 & 0.5 \\ 0.5 & -0.5 & 0 \end{bmatrix}$
- (b) $A^{-1} = \begin{bmatrix} 0 & -0.5 & 0.5 \\ -0.5 & 0 & 0.5 \\ 0.5 & 0.5 & 0 \end{bmatrix}$
- (c) $A^{-1} = \begin{bmatrix} 0.5 & -0.5 & 0 \\ -0.5 & 0 & 0.5 \\ 0.5 & 0.5 & 0 \end{bmatrix}$
- (d) the inverse does not exist

4. ☐ MULTI ☐ Single

Find the kernel of $A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$.

- (a) $\ker(A) = \text{span} \left\{ \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix} \right\}$
- (b) $\ker(A) = \text{span} \left\{ \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} \right\}$
- (c) $\ker(A) = \text{span} \left\{ \begin{bmatrix} -2 \\ 1 \\ -1 \end{bmatrix} \right\}$
- (d) $\ker(A) = \text{span} \left\{ \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} \right\}$

5. ☐ MULTI ☐ Single

An $n \times k$ matrix A has the following kernel:

$$\ker(A) = \text{span} \left\{ \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} \right\}$$

What is the dimension of its image $\dim(\text{im}(A))$?

- (a) $\dim(\text{im}(A)) = n - 3$
- (b) $\dim(\text{im}(A)) = k - 1$
- (c) $\dim(\text{im}(A)) = k - 2$
- (d) $\dim(\text{im}(A)) = n$

6. ☐ MULTI ☐ Single

Given the matrix $A = B \cdot C \cdot D$ find its inverse A^{-1} .

- (a) $A^{-1} = C^{-1}D^{-1}B^{-1}$
- (b) $A^{-1} = D^{-1}B^{-1}C^{-1}$
- (c) $A^{-1} = D^{-1}C^{-1}B^{-1}$
- (d) $A^{-1} = B^{-1}C^{-1}D^{-1}$

7. ☐ MULTI ☐ Single

A linear map $D : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is given by the following matrix in the standard basis:

$D_{\text{st}} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$. How is the map represented in the following basis: $\left\{ \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right\}$?

- (a) $D_{\text{new}} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$
- (b) $D_{\text{new}} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

$$(c) D_{\text{new}} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$(d) D_{\text{new}} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

8. ☐ MULTI ☐ Single

Consider the standard basis in $\mathbb{R}^3$: $\{e_x, e_y, e_z\}$.

Which of the following matrices represents a counterclockwise rotation around the z -axis?

$$(a) \mathcal{R} = \begin{bmatrix} \cos \varphi & 0 & \sin \varphi \\ 0 & 1 & 0 \\ \sin \varphi & 0 & \cos \varphi \end{bmatrix}$$

$$(b) \mathcal{R} = \begin{bmatrix} 1 & 0 & -\sin \varphi \\ \cos \varphi & 1 & \cos \varphi \\ \sin \varphi & 0 & 1 \end{bmatrix}$$

$$(c) \mathcal{R} = \begin{bmatrix} \cos \varphi & -\sin \varphi & 0 \\ \sin \varphi & \cos \varphi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$(d) \mathcal{R} = \begin{bmatrix} \cos \varphi & -\sin \varphi & 0 \\ \sin \varphi & \cos \varphi & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

9. ☐ MULTI ☐ Single

Consider the space of 2×2 Hermitian Matrices $H_2(\mathbb{C})$ (the space of 2×2 matrices A with complex entries such that $A^\dagger := \overline{A}^T = A$).

Which of the following is true?

- (a) $H_2(\mathbb{C})$ is a vector space over the field of complex numbers, but not over the real numbers.
- (b) $H_2(\mathbb{C})$ is not a vector space over the field of real numbers or complex numbers.
- (c) $H_2(\mathbb{C})$ is a vector space over the field of real numbers, but not over the complex numbers.
- (d) $H_2(\mathbb{C})$ is a vector space over the field of real numbers and over the complex numbers.

10. ☐ MULTI ☐ Single

Consider the vector space $P_2(\mathbb{R}) = \{p(x) \mid p(x) \text{ is a quadratic polynomial}\}$.

Is the derivative operator $\mathcal{D} : p(x) \mapsto p'(x) \equiv \frac{d}{dx}p(x)$ a linear operator? If it is, how is it represented in the standard basis $\mathfrak{B} = \{1, x, x^2\}$

Hint: You can express a polynomial $ax^2 + bx + c$ as $\begin{bmatrix} c \\ b \\ a \end{bmatrix}$

- (a) $\mathcal{D}$ is a linear operator with $[\mathcal{D}]_{\mathfrak{B}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{bmatrix}$
- (b) $\mathcal{D}$ is not a linear operator

- (c) $\mathcal{D}$ is a linear operator with $[\mathcal{D}]_{\mathfrak{B}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$
- (d) $\mathcal{D}$ is a linear operator with $[\mathcal{D}]_{\mathfrak{B}} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Total of marks: 10