

Week 7: The determinant and eigenvalues

- (1) MULTIPLE CHOICE One answer only

Consider the linear transformation given by the matrix.

$$T = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 4 & 5 & 6 \\ 3 & 4 & 5 & 6 & 7 \\ 4 & 5 & 6 & 7 & 8 \end{bmatrix}$$

Is T invertible? Why? (Hint: Pay attention to the first three rows.)

- a. T is invertible because $\det(T) = 0$
- b. T is not invertible because $\det(T) \neq 0$
- c. T is invertible because $\det(T) \neq 0$
- d. T is not invertible because $\det(T) = 0$

- (2) MULTIPLE CHOICE One answer only

Consider the linear transformation $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ that has the following values on the basis elements:

$$T : (1, 0, 0) \mapsto (-1, 0, 1)$$

$$T : (0, 1, 0) \mapsto (3, -2, -1)$$

$$T : (0, 0, 1) \mapsto (1, 1, 1)$$

Is T invertible? Why?

- a. T is invertible because $\det(T) = 0$
- b. T is not invertible because $\det(T) = 0$
- c. T is invertible because $\det(T) \neq 0$
- d. T is not invertible because $\det(T) \neq 0$

- (3) MULTIPLE CHOICE One answer only

Use Cramer's rule to solve for y in

$$A\vec{x} = \begin{bmatrix} 2 & 6 & 2 \\ 1 & 4 & 2 \\ 5 & 9 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \vec{b}.$$

- a. 2
- b. -1
- c. 1
- d. -2

- (4) MULTIPLE CHOICE One answer only

Use Cramer's rule to solve for y in

$$ax + by + cz = 1$$

$$dx + ey + fz = 0$$

$$gx + hy + iz = 0.$$

You can assume that the relevant 3×3 matrix has non-zero determinant D .

- a. $\frac{ei - fh}{D}$
- b. $\frac{fh - ei}{D}$
- c. $\frac{fg - di}{D}$
- d. $\frac{di - fg}{D}$

(5) MULTIPLE CHOICE One answer only

Use Cramer's rule to solve for x_1 in

$$\begin{aligned} 2x_1 + x_2 &= 1 \\ x_1 + 2x_2 + x_3 &= 0 \\ x_2 + 2x_3 &= 0. \end{aligned}$$

- a. $\frac{1}{2}$
- b. $\frac{3}{4}$
- c. $\frac{3}{4}$
- d. $\frac{1}{4}$

(6) MULTIPLE CHOICE One answer only

Compute the inverse of the matrix (try using the classical adjoint)

$$H = \begin{bmatrix} 1 & a & c \\ 0 & 1 & b \\ 0 & 0 & 1 \end{bmatrix}$$

where a, b, c are arbitrary real numbers.

- a. $\begin{bmatrix} 1 & 0 & 0 \\ -a & 1 & 0 \\ ab - c & -b & 1 \end{bmatrix}$
- b. $\begin{bmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ c - ab & b & 1 \end{bmatrix}$
- c. $\begin{bmatrix} 1 & -a & ab - c \\ 0 & 1 & -b \\ 0 & 0 & 1 \end{bmatrix}$
- d. $\begin{bmatrix} 1 & a & c - ab \\ 0 & 1 & b \\ 0 & 0 & 1 \end{bmatrix}$

(7) MULTIPLE CHOICE One answer only

Compute the inverse of the matrix (try using the classical adjoint)

$$R_\theta = \begin{bmatrix} \cos(\theta) & 0 & \sin(\theta) \\ 0 & 1 & 0 \\ -\sin(\theta) & 0 & \cos(\theta) \end{bmatrix}$$

where θ is an arbitrary real number.

- a. $R_{\theta-1}$
- b. $-R_\theta$
- c. $-R_\theta$
- d. $R_{-\theta}$

(8) MULTIPLE CHOICE One answer only

Find the eigenvalues and the associated eigenvectors of the matrix

$$A = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}.$$

- a. $\lambda_1 = \sqrt{2}, v_1 = (1, \sqrt{2} - 1)$
 $\lambda_2 = -\sqrt{2}, v_2 = (1, -\sqrt{2} - 1)$
- b. $\lambda_1 = 1, v_1 = (\sqrt{2} - 1, 1)$
 $\lambda_2 = -1, v_2 = (\sqrt{2} + 1, 1)$
- c. $\lambda_1 = 1, v_1 = (1, \sqrt{2} - 1)$
 $\lambda_2 = -1, v_2 = (1, -\sqrt{2} - 1)$
- d. $\lambda_1 = \sqrt{2}, v_1 = (1, 1)$
 $\lambda_2 = -\sqrt{2}, v_1 = (1, -1)$

(9) MULTIPLE CHOICE One answer only

Find the eigenvalues and the associated eigenvectors of the matrix

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}.$$

- a. $\lambda_1 = 0, v_1 = (-2, -1)$
 $\lambda_2 = 5, v_2 = (-1, 2)$
- b. $\lambda_1 = 0, v_1 = (2, 1)$
 $\lambda_2 = 3, v_2 = (1, 3)$
- c. $\lambda_1 = 0, v_1 = (2, -1)$
 $\lambda_2 = 5, v_2 = (1, 2)$
- d. $\lambda_1 = 1, v_1 = (2, 1)$
 $\lambda_2 = 5, v_2 = (1, 2)$

(10) MULTIPLE CHOICE One answer only

Find all eigenvalues and associated eigenvectors of the matrix

$$A = \begin{bmatrix} 7 & 0 & -3 \\ -9 & -2 & 3 \\ 18 & 0 & -8 \end{bmatrix}.$$

- a. $\lambda_1 = \lambda_2 = -2, v = (t, s, 3t)$ for $s, t \in \mathbb{R}$ and $\lambda_3 = 1, v_3 = (1, -1, 2)$
- b. All eigenvalues are zero.
- c. $\lambda_1 = \lambda_2 = 1, v = (1, 0, 3)$ and $\lambda_3 = 2, v_3 = (1, -1, 2)$
- d. $\lambda_1 = \lambda_2 = -2, v = (1, 0, 3)$ and $\lambda_3 = 1, v_3 = (1, -1, 2)$

Total of marks: 10