

Week 9: Diagonalization and Normal Matrices

- (1) MULTIPLE CHOICE One answer only

Is the matrix

$$A = \begin{bmatrix} 6 & -1 \\ 2 & 3 \end{bmatrix}$$

diagonalizable?

- a. A is diagonalizable
- b. Information given is insufficient
- c. A is not diagonalizable

- (2) MULTIPLE CHOICE One answer only

Is the matrix

$$A = \begin{bmatrix} -2 & -4 & 2 \\ -2 & 1 & 2 \\ 4 & 2 & 5 \end{bmatrix}$$

diagonalizable?

- a. Information given is insufficient
- b. A is diagonalizable
- c. A is not diagonalizable

- (3) MULTIPLE CHOICE One answer only

Let $0 < \mu < 1$ and suppose an $n \times n$ matrix B has characteristic equation

$$\lambda(\lambda - \mu)(\lambda - \mu^2) \cdots (\lambda - \mu^{n-1}).$$

Is B diagonalizable?

- a. Information given is insufficient
- b. B is diagonalizable
- c. B is not diagonalizable

- (4) MULTIPLE CHOICE One answer only

Consider the vector space $\mathcal{P}_n$ of polynomials of degree n and consider the linear map $D : \mathcal{P}_n \rightarrow \mathcal{P}_n$ that maps each polynomial to its derivative. First compute the matrix of D in the standard basis $\{1, x, x^2, \dots, x^n\}$ (you can refer to a previous exercise here). If $n > 0$, is D diagonalizable?

- a. D is diagonalizable
- b. D is not diagonalizable
- c. Information given is insufficient

- (5) MULTIPLE CHOICE One answer only

Is the matrix

$$B = \begin{bmatrix} 0 & -i & i \\ i & 0 & -i \\ -i & i & 0 \end{bmatrix}$$

diagonalizable?

- a. Information given is insufficient
- b. B is not diagonalizable
- c. B is diagonalizable

(6) MULTIPLE CHOICE One answer only

Find all values of k that make the matrix

$$A = \begin{bmatrix} 1 & k & 0 \\ 0 & 1 & k \\ 0 & 0 & 2 \end{bmatrix}$$

diagonalizable.

- a. A is diagonalizable only when $k = 0$.
- b. A is diagonalizable only when $|k| = 1$.
- c. A is diagonalizable only when $k \neq 0$.
- d. A is diagonalizable only when $|k| \neq 1$.

(7) MULTIPLE CHOICE One answer only

Consider the 2×2 matrix

$$A = \begin{bmatrix} 6 & -1 \\ 2 & 3 \end{bmatrix}.$$

Find a formula for A^k .

- a. $\begin{bmatrix} 5^k & 0 \\ 0 & 4^k \end{bmatrix}$
- b. $\begin{bmatrix} 2 \cdot 5^k - 4^k & 5^k + 4^k \\ 2 \cdot 5^k - 4^k & -5^k + 2 \cdot 4^k \end{bmatrix}.$
- c. $\begin{bmatrix} 2 \cdot 5^k - 4^k & -5^k + 4^k \\ 2 \cdot 5^k - 2 \cdot 4^k & -5^k + 2 \cdot 4^k \end{bmatrix}.$
- d. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(8) MULTIPLE CHOICE One answer only

Suppose A is an $n \times n$ matrix with distinct eigenvalues $\lambda_1, \dots, \lambda_n$. Suppose that for all k , $|\lambda_k| < 1$. Compute the limits $\lim_{p \rightarrow \infty} A^p$ and $\lim_{p \rightarrow \infty} \exp(A^p)$.

- a. $\lim_{p \rightarrow \infty} A^p = I, \lim_{p \rightarrow \infty} \exp(A^p) = 0$
- b. $\lim_{p \rightarrow \infty} A^p = I, \lim_{p \rightarrow \infty} \exp(A^p) = I$
- c. $\lim_{p \rightarrow \infty} A^p = 0, \lim_{p \rightarrow \infty} \exp(A^p) = I$
- d. $\lim_{p \rightarrow \infty} A^p = 0, \lim_{p \rightarrow \infty} \exp(A^p) = 0$

(9) MULTIPLE CHOICE One answer only

Recall that when x, y are real numbers, we have the identity $\exp(x) \exp(y) = \exp(x + y)$. In this exercise we will investigate whether this holds for matrices. Consider the matrices

$$X = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \quad \text{and} \quad Y = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}.$$

Compute $\exp(X) \exp(Y)$ and $\exp(X + Y)$.

- a. The matrix exponential is undefined for $\exp(Y)$.
- b. $\exp(X)\exp(Y) = \exp(X+Y) = \begin{bmatrix} e & e^2 - e \\ 0 & e^2 \end{bmatrix}$
- c. $\exp(X)\exp(Y)\exp(X+Y) = \begin{bmatrix} e & e \\ 0 & e^2 \end{bmatrix}$
- d. $\exp(X)\exp(Y) = \begin{bmatrix} e & e \\ 0 & e^2 \end{bmatrix}, \exp(X+Y) = \begin{bmatrix} e & e^2 - e \\ 0 & e^2 \end{bmatrix}$

(10) MULTIPLE CHOICE One answer only

Consider the matrix

$$A = \begin{bmatrix} 1 & -2 - i & 4\exp(i\frac{\pi}{4}) \\ 1 + i & i & 2 - 7i \end{bmatrix}$$

What is $A^\dagger$?

- a. $\begin{bmatrix} 1 & 1 - i \\ -2 + i & -i \\ 4\exp(-i\frac{\pi}{4}) & 2 + 7i \end{bmatrix}$
- b. $\begin{bmatrix} 1 & 1 + i \\ -2 - i & i \\ 4\exp(i\frac{\pi}{4}) & 2 - 7i \end{bmatrix}$
- c. $\begin{bmatrix} 1 & -2 - i & 4\exp(i\frac{\pi}{4}) \\ 1 + i & i & 2 - 7i \end{bmatrix}$
- d. $\begin{bmatrix} 1 & -2 + i & 4\exp(-i\frac{\pi}{4}) \\ 1 - i & -i & 2 + 7i \end{bmatrix}$

Total of marks: 10