Constructor University — October 24, 2025
Fall 2025

Stochastic Modeling and Financial Mathematics

Quiz 4

Name: ____

1. **(7 points)** In class, we discussed possible calibrations of Binomial Tree models. For the rate of return y of a stock, we found that the average is given by

$$\mathbb{E}(y) = \left(\ln \frac{u}{d}\right) np + n \ln d,$$

and the variance by

$$\mathrm{Var}(y) = \left(\ln \frac{u}{d}\right)^2 np(1-p).$$

Instead of the condition $ud = 1$ we discussed in class, let us now require that $p = \frac{1}{2}$. Show that the choice

$$u = e^{\mu \frac{T}{n} + \sigma \sqrt{\frac{T}{n}}},$$
$$d = e^{\mu \frac{T}{n} - \sigma \sqrt{\frac{T}{n}}}$$

also leads to the correct average and variance, i.e., show that for that choice $\mathbb{E}(y) = \mu T$ and $Var(y) = \sigma^2 T$.

2. **(7 points)** In class we discussed the Black–Scholes formula

$$C = S\Phi(x) - Ke^{-rT}\Phi(x - \sigma\sqrt{T}),$$

with

$$x = \frac{\ln\frac{S}{K} + (r + \frac{\sigma^2}{2})T}{\sigma\sqrt{T}}.$$

Explain this formula, i.e., explain what all the variables and functions are, and very briefly (ideally in one sentence) how we arrived at this formula.

3. **(6 points)** The following function returns the option price of a European option. The inputs are the payoff function, and the variables K, n, r, $sigma$, t. Note that in the "for loop" the correct stock price at step n is additionally computed. Modify the function so it correctly computes the values of an American option. *Hint: Note that the code below is correct. Most efficiently, the exercise can be solved by modifying one line of code.*

```
def binomial_european (payoff,K,n,r,sigma,t):
    R = exp(r*t/n)
    u = exp(sigma*sqrt(t/n))
    d = 1.0/u
    p = (R-d)/(u-d)
    q = 1-p

    S = d**arange(n+1) * u**arange(n,-1,-1)
    C = empty((n+1,n+1))
    C[:,-1] = payoff(S,K)

    for i in range(n-1,-1,-1):
        S = S[:i+1]/u
        C[:i+1,i] = (p*C[:i+1,i+1] + q*C[1:i+2,i+1])/R

    return C, d, u
```