Constructor University
Fall 2025

November 7, 2025

Stochastic Modeling and Financial Mathematics

Quiz 5

Name: ____

1. **(5 points)** Consider a random variable X that is distributed according to a standard normal distribution (i.e., with expectation value 0 and variance 1). Then consider two other normally distributed independent random variables X_1 and X_2 such that $X = X_1 + X_2$ (now the expectation and variance of X_1 might be different). What are the expectation value and variance of X_1? Show the steps of the computation. How does this result generalize to $X = X_1 + \ldots + X_n$?

2. **(5 points)** In class, we discussed two types of stochastic integrals. Provide the names of each integral type. Then, for each integral type, state what the result of $\int_0^T W(t)\,\mathrm{d}W(t)$ is, were $W(t)$ is a Brownian motion. (Just the result, no computation necessary.)

3. **(5 points)** We consider the stochastic differential equation

$$X(t) = X(0) + \int_0^t f(X(s), s)\,\mathrm{d}s + \int_0^t g(X(s), s)\,\mathrm{d}W(s),$$

with $W(s)$ a Brownian motion. Write down the Euler-Maruyama method for this equation.

4. **(5 points)** We consider the following python program.

```
from pylab import *

T = 1.0
N = 600
dt = T/N
M = 4000

dW = normal(0, 1, (M,N))*sqrt(dt)
W = c_[zeros(M), cumsum(dW, axis=1)]

Wmean = mean(W, axis=0)
Wstd = std(W, axis=0)
Wplot = W[:3,:].T

t = linspace(0,T,N+1)
figure()
plot (t,Wmean, 'r--',
      t,Wmean+Wstd, 'r',
      t,Wplot, 'b-',
      t,Wmean-Wstd, 'r')
show()
```

Draw the output of this program.