

2. Options and Binomial Tree Models

2.1 Option Basics

Option: Contract (or financial instrument) that depends on the future price of an underlying asset (most commonly stocks $\rightarrow$ our focus).

Such instruments are called "derivatives" ("derivative financial instruments")

Option contract: Right to buy or sell underlying asset at a "strike price" K at or before "expiration date" T .

Types of options:

Buy/sell: • Call option: right to buy asset
• Put option: right to sell asset

At / at or before: • European: can be exercised only at expiration
• American: can be exercised at or before expiration

Def.: • price of underlying asset $S(t)$
• payoff = value of option at expiration

Ex.: European call, $K = 100 \text{ EUR}$, $T = 1 \text{ year}$, suppose $S(0) = 90 \text{ EUR}$

Suppose at $T = 1 \text{ year}$: $S(T) = 120 \text{ EUR}$

↗ 1 stock worth 120 EUR
(bought it for 100 EUR = K only)

$\Rightarrow$ Exercise option, with payoff $S(T) - K = 20 \text{ EUR}$

Suppose $S(T) = 70 \text{ EUR} \Rightarrow$ do not exercise option, payoff = 0 EUR

nothing

Option contracts have a price P .

Q.: How expensive should this contract be? What should P be?

Remark: P will depend on the model for $S(t)$ (how price of underlying asset evolves).

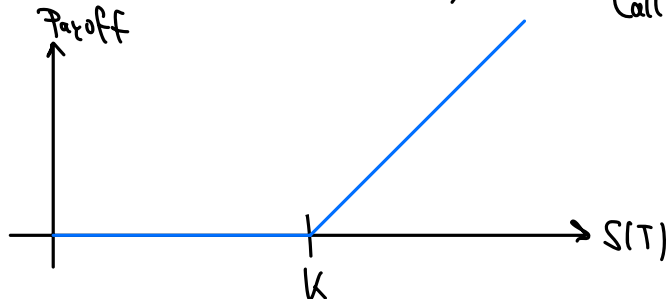
What are options good for? $\rightarrow$ Insurance, speculation

Basic assumption: No risk-free profit.

Called: No arbitrage assumption

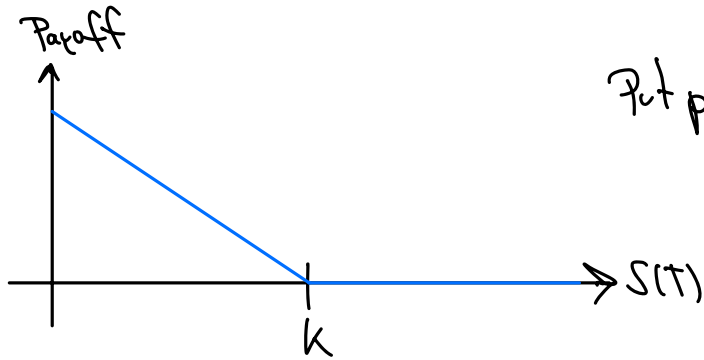
Payoff is a fct. of K and $S(T)$:

Call payoff:



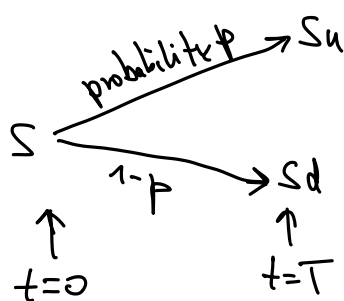
$$\text{Call payoff} = \max\{S(T) - K, 0\}$$

Put payoff



$$\text{Put payoff} = \max\{K - S(T), 0\}$$

2.2 Binary Model



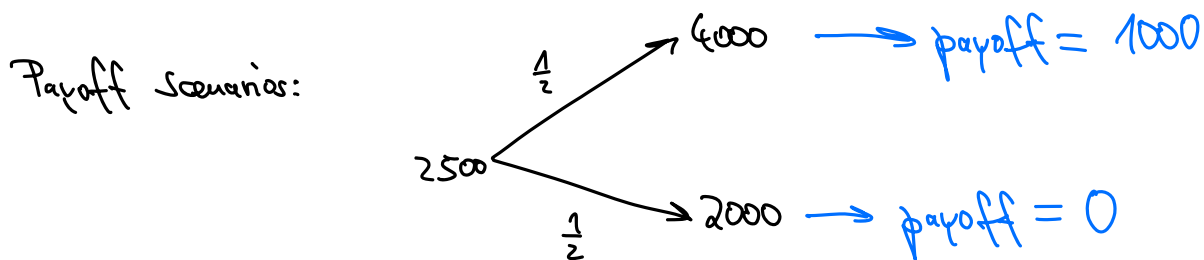
here: $u > 1$
 $d < 1$

} asset model

↳ parameters p, u, d

Example: $S = 2500 \text{ EUR}$, $S_u = 4000 \text{ EUR}$, $p = 0.5$
 $S_d = 2000 \text{ EUR}$

Option: European call, $K = 3000 \text{ EUR}$ strike price



What should be option price P ?

Guess: Average payoff given the probability p , i.e., $0.5 \cdot 1000 + 0.5 \cdot 0 = 500$

Strategy 1 for seller: $T=0$: Make 500 EUR

$T=1$:
up: buy stock for 4000 EUR, sell it to option holder for 3000 EUR (payoff = 1000 EUR)
In total: Cost 500 EUR

down: In total: Made 500 EUR

Strategy 2:

$T=0$ Make 500 EUR, borrow 2000 EUR to buy the stock for 2500 EUR

$T=1$
up (4000): Sell stock for 3000 EUR
 $\Rightarrow$ In total: Made 1000 EUR
down (2000): Sell stock for 2000 EUR $\Rightarrow$ Made: 0 EUR

Strategy 2 would lead to riskless profit!

We analyze such strategies in general next time.