Analytical Mechanics Session 1

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Lecture notes from Fall 2026

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Information:
- website, handwritten notes
- Schupp lecture notes
- homework submission on moodle
- discussions + announcements on Teams

About this class:

- We all know Newton's laws from high school
- This class offers more: new formalisms that reveal more structure and symmetry: Lagrange and Hamilton
- Topics:
  - Newtonian Mechanics (one and several particles, rigid bodies, conservation laws, reference frames, rotations, two-body problem) $\sim \frac{1}{3}$
  - Lagrange $\sim \frac{1}{3}$
  - Hamilton $\sim \frac{1}{3}$
  - some Special Relativity

0. Introduction

Classical Mechanics:

- one of the foundational theories of physics
- with gravity: first "fully closed" theory of the world

Statistical Mechanics (k_B)

Classical Mechanics | Electrodynamics (c) | Quantum Mechanics ($\hbar$)

↳ Newtonian Gravity (G)

↳ Special Relativity (c)

("Classical") General Relativity → ? ← Quantum Field Theory

1. Geometry

Classical Mechanics is formulated in Euclidean space and with absolute time:

- positions $\vec{r} = \begin{pmatrix} r_1 \\ r_2 \\ r_3 \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3$
- time $t \in \mathbb{R}$

Note: special relativity: spacetime: $X = (ct, \vec{r}) \in \mathbb{R}^4$

coordinate $\vec{r}_A$ coordinate $\vec{r}_B$

Euclidean distance $\overline{AB}$ between two points A and B:

$$(\overline{AB})^2 = |\underbrace{\vec{r}_A - \vec{r}_B}_{=:\Delta\vec{r}}|^2 = \Delta\vec{r} \cdot \Delta\vec{r} = \sum_{i,j=1}^{3} \delta_{ij}\, \Delta r_i\, \Delta r_j$$

Recall: • dot/scalar product: $\vec{r} \cdot \vec{s} = \sum_{i=1}^{3} r_i s_i = r_1 s_1 + r_2 s_2 + r_3 s_3$

• Kronecker delta: $\delta_{ij} = \begin{cases} 1 & \text{if } i=j \\ 0 & \text{else} \end{cases}$

$= (I)_{ij}$, I = identity matrix

Note: We can define a different metric through $(\overline{AB})^2 = \sum_{i,j=1}^{3} g_{ij}\, \Delta r_i\, \Delta r_j$

$g_{ij} = g_{ji}$ "metric tensor"

Recall $\vec{r} \cdot \vec{s} = |\vec{r}|\,|\vec{s}| \cos\varphi$, φ = angle between $\vec{r}$ and $\vec{s}$, so different metrics def. different geometries.

Another useful "tensor": Levi-Civita symbol $\varepsilon_{ijk} = \begin{cases} +1 & \text{for } (i,j,k) = (1,2,3) \text{ or } (2,3,1) \text{ or } (3,1,2) \\ -1 & \text{for } (i,j,k) = (3,2,1) \text{ or } (2,1,3) \text{ or } (1,3,2) \\ 0 & \text{else} \end{cases}$

More generally: $\varepsilon_{\mu_1 \dots \mu_n} = \begin{cases} +1 & \text{if } \mu_1, \dots, \mu_n \text{ even permutation of } 1, \dots, n \\ -1 & \text{if } \mu_1, \dots, \mu_n \text{ odd permutation of } 1, \dots, n \\ 0 & \text{else} \end{cases}$

$= $ totally antisymmetric object with $\varepsilon_{1\ldots n} = +1$

Totally antisym.: $\varepsilon_{\ldots\mu_i\mu_j\ldots} = -\varepsilon_{\ldots\mu_j\mu_i\ldots}$ (swapping any two (adjacent) indices gives minus sign)

Notes:

- Property: $\sum_{i=1}^{3} \varepsilon_{ijk}\varepsilon_{ilm} = \delta_{jl}\delta_{km} - \delta_{jm}\delta_{kl}$

  or more generally: $\sum_{\mu_1,\ldots,\mu_p} \varepsilon_{\mu_1\ldots\mu_p\alpha_1\ldots\alpha_{n-p}}\varepsilon_{\mu_1\ldots\mu_p\beta_1\ldots\beta_{n-p}} = p!(n-p)!\,\overbrace{\delta^{[\alpha_1}_{\beta_1}\cdots\delta^{\alpha_{n-p}]}_{\beta_{n-p}}}^{\text{antisym. over all indices}}$

- In $n=3$: cross product $\vec{r}\times\vec{s} = \sum_{i,j,k=1}^{3} \varepsilon_{ijk} r_j s_k \underbrace{\hat{e}_i}$

  Euclidean basis vector in direction i:

  $\hat{e}_i = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \leftarrow$ i-th coordinate