Stochastic Modeling and Financial Mathematics — Session 2

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Lecture notes from Fall 2026

Git setup for this class:

- public course repo: spetrat/sml_2026.git
- student: own private repo:
  - use public course repo as "remote" to pull from
  - do homework in own repo
  - Instructor and TA have "write" access
- Workflow for students:
  - "git pull" of class repo
  - "git push" to own repo
  - do your work, "git add" and "git commit" + git push
  - "git pull"

Setup steps:
- download + install git
- configure git (use command line)
- get github account
- set up ssh key to access github
- locally:
  - create own private repo
  - clone course repo
  - configure remote
  - need to give TA and instructor "write access"

0.2 Scientific Python

- designed to be fast and efficient, uses $\underbrace{\text{numpy arrays and vector operations}}_{\text{runs near machine speed, even though interpreted language}}$

ToDo:

- Install python programming environment

Recommendation for class: Anaconda python
↳ spyder editor
↳ jupyter notebooks

HW: submit ".py" files only

1. Basics of Financial Math

1.1 Time value of money

r = annual interest rate, PV = present value, FV = future value

The value of money changes over time:

$$FV_n = PV(1+r)^n = \text{future value of } PV \text{ after } n \text{ years}$$

$\uparrow$ n years

$$\Longleftrightarrow PV = FV_n (1+r)^{-n}$$

If interest is compounded several times, say m times a year, then

$$FV = PV\left(1+\frac{r}{m}\right)^{nm}$$

Terminology:
- BEY ("bond equivalent yield")
  ↳ semiannually (twice a year, $m=2$)
- MEY ("mortgage equivalent yield")
  ↳ monthly ($m=12$)

How do you compare different compounding standards?

$\Rightarrow$ effective interest rate r_{eff}:

$$\left(1+\frac{r}{m}\right)^{nm} = (1+r_{eff})^n$$

$$\Rightarrow 1+r_{eff} = \left(1+\frac{r}{m}\right)^m \Rightarrow \boxed{r_{eff} = \left(1+\frac{r}{m}\right)^m - 1}$$

Ex.: $r = 10\%$, BEY

$$\Rightarrow r_{eff} = \left(1 + \frac{0.1}{2}\right)^2 - 1$$

$$= 1.05^2 - 1 = (1 + 0.05)^2 - 1$$

$$= 1 + 0.1 + 0.0025 - 1$$

$$= 0.1025 = 10.25\%$$

If $m > 1$, is $r_{eff} > r$?

$$r_{eff} = \underbrace{\left(1 + \frac{r}{m}\right)^m} - 1 = r + \text{rest} > r$$

Yes

$$= 1 + m\frac{r}{m} + \underbrace{\text{rest}}_{>0}$$

(More generally: $(1+x)^m \geq 1 + mx$ (Bernoulli's inequality) for all $x > -1$)

One useful idealization: continuous interest compounding: $m \to \infty$

$$(1 + \frac{x}{n})^n \xrightarrow[n\to\infty]{} e^x$$

Then: $$FV = \lim_{m\to\infty} PV\left(1 + \frac{r}{m}\right)^{mn}$$

$$= PV\left(\lim_{m\to\infty}\left(1 + \frac{r}{m}\right)^m\right)^n$$

$$= PV\left(e^r\right)^n = PV\, e^{rn}$$