Constructor University
Spring 2026

February 16, 2026

Elements of Calculus

Homework 2 (covering Weeks 3 and 4)

Due on March 4, 2026, before the tutorial! Please submit on moodle.

Problem 1 [2 points]
Prove that the function

$$f(x) = \begin{cases} 0 & \text{, for } x = 0 \\ \sin\left(\frac{1}{x}\right) & \text{, for } x \neq 0 \end{cases}$$

is not continuous at $x = 0$ by constructing a sequence $(x_n)_{n\geq 0}$ that converges to 0, but for which $f(x_n)$ does not converge to $f(0)$.

Problem 2 [2 points]
Suppose that for some $x_0 \in \mathbb{R}$ the sequence

$$x_n = x_0 - \frac{1}{n}$$

is given. Now consider two functions $f(x)$ and $g(x)$ (from $\mathbb{R}$ to $\mathbb{R}$). Suppose that

$$\lim_{n\to\infty} f(x_n) = f(x_0),$$

but

$$\lim_{n\to\infty} g(x_n) \neq g(x_0).$$

What can be deduced about the continuity of f and g at x_0?

Problem 3 [3 points]
Using the definition of limits of functions via converging sequences, prove that for

$$f(x) = \begin{cases} 0 & \text{, for } x = 0 \\ \frac{x}{|x|} & \text{, for } x \neq 0 \end{cases}$$

we have $\lim_{x\to 0^+} f(x) = 1$, $\lim_{x\to 0^-} f(x) = -1$, and that $\lim_{x\to 0} f(x)$ does not exist.

Problem 4 [4 points]
Let

$$f(x) = \frac{x^2 + 3x - 10}{x - 2}.$$

What is the domain of f? What is $\lim_{x\to 2} f(x)$?

Problem 5 [3 points]

In class we discussed the product rule $(fg)' = f'g + fg'$. Generalize this rule for the n-th derivative $(fg)^{(n)}$.

Problem 6 [6 points]

Consider the curve parametrized by φ where $x = a\cos(3\varphi)$ and $y = a\cos(\varphi)$ for some given $a \in \mathbb{R}$.

(a) Calculate $\frac{dy}{dx}$ using the given parametrization.

(b) Show that the curve satisfies $4y^3 = a^2(x + 3y)$.

(c) Calculate $\frac{dy}{dx}$ by implicit differentiation of the equation from (b). Does the result coincide with (a)?