Constructor University
Spring 2026

February 2, 2026

Linear Algebra

Homework 1

Due on February 11, 2026, before the tutorial!

Problem 1 [**5 points**]

Let $W \subset V$ be a subspace. Show that $(V \setminus W) \cup \{0\}$ is a subspace of V if and only if $W = \{0\}$ or $W = V$.

Problem 2 [**5 points**]

(a) Let V be a vector space. Prove that for any subset $S \subset V$, $\mathrm{span}(S)$ is a subspace.

(b) Prove that the span of S is equal to the intersection of all subspaces of V that contain S.

Problem 3 [**5 points**]

Consider the space of real $n \times n$ matrices. Which of the following matrices form a subspace? Give a brief and concise proof for each case.

(a) orthogonal matrices

(b) invertible matrices

(c) matrices with trace zero

(d) symmetric matrices

(e) antisymmetric matrices

Problem 4 [**5 points**]

Prove that $\mathbb{R}$ as a vector space over $\mathbb{Q}$ is infinite dimensional.