Constructor University February 9, 2026
Spring 2026

Linear Algebra

Homework 2

Due on February 18, 2026, before the tutorial.

Problem 1 [6 points]

(Axler, Ch. 2, Exercise 7.) Prove that a vector space V is infinite dimensional if and only if there is a sequence $v_1, v_2, \ldots$ of vectors in V such that $(v_1, \ldots, v_n)$ is linearly independent for every positive integer n.

Problem 2 [6 points]

Prove that $\mathbb{R}$ as a vector space over $\mathbb{Q}$ is infinite dimensional.

Problem 3 [5 points]

(Kostrikin, Manin, Ch. 2, Exercise 4.) Calculate the dimensions of the following spaces:

(a) the space of polynomials of degree $\leq p$ of n variables;

(b) the space of homogeneous polynomials of degree p of n variables; (A homogeneous polynomial is a polynomial whose nonzero terms all have the same degree.)

(c) the space of functions in $F(S)$, $|S| < \infty$ that vanish at all points of the subset $S_0 \subset S$.

Problem 4 [3 points]

(Axler, Ch. 2, Exercise 8.) Let U be the subspace of $\mathbb{R}^5$ defined by $U = \{(x_1, x_2, x_3, x_4, x_5) \in \mathbb{R}^5 : x_1 1 = 3x_2 and x_3 = 7x_4\}$. Find a basis of U.