# Linear Algebra

## Homework 6

Due on March 18, 2026, before the tutorial.

**Problem 1 [5 points]**

Let $M, N \subset L$ be subspaces. Prove that the mapping

$$(M + N)/N \to M/(M \cap N), m + n + N \mapsto m + M \cap N$$

is a linear isomorphism.

**Problem 2 [5 points]**

Prove that the canonical mapping

$$M \to (M \oplus N)/N, m \mapsto m + N$$

is an isomorphism.

**Problem 3 [5 points]**

Let $V_1, \ldots, V_n, V_1', \ldots, V_n'$ be subspaces of a vector space $V$. We say $V_1, \ldots, V_n$ and $V_1', \ldots, V_n'$ are *identically arranged* if there is a linear isomorphism $f : V \to V$ such that $f(V_i) = V_i'$.

(a) Consider $n = 1$. Prove that $V_1$ and $V_1'$ are identically arranged if and only if $\dim V_1 = \dim V_1'$.

(b) Consider $n = 2$. Prove that $V_1, V_2$ and $V_1', V_2'$ are identically arranged if and only if $\dim V_1 = \dim V_1'$, $\dim V_2 = \dim V_2'$, and $\dim V_1 \cap V_2 = \dim V_1' \cap V_2'$.

**Problem 4 [5 points]**

Suppose
$$U = \{(x_1, x_2, \ldots) \in F^\infty : x_j \neq 0 \text{ for only finitely many } j\}.$$

(a) Show that $U$ is a subspace of $F^\infty$.

(b) Prove that $F^\infty/U$ is infinite dimensional.