Constructor University April 27, 2026
Spring 2026

Linear Algebra

Homework 11

Due on May 6, 2026, before the tutorial.

Problem 1 [4 points]

Let L be the vector space of real differentiable functions on the interval $(0, 1)$.

(a) Prove that $g : L \times L \to \mathbb{R}$ defined by $g(f, g) = (fg)'(0)$ is a symmetric bilinear form.

(b) Determine $\ker g$.

Problem 2 [4 points]

Let g be a symmetric bilinear form given by the Gram matrix

$$\begin{pmatrix} 3 & -2 & 0 \\ -2 & 2 & -2 \\ 0 & -2 & 1 \end{pmatrix}.$$

(a) Find a basis A such that the Gram matrix in this basis is diagonal.

(b) Find a basis B such that the Gram matrix is

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}.$$

Problem 3 [6 points]

For the following cases, give examples of inner product spaces with the specified symmetry and signature. Also write down at least two different orthogonal decompositions of the space and how the inner product acts on each of the spaces in the decomposition:

(a) 2-dimensional non-degenerate symplectic over $\mathbb{R}$,

(b) 3-dimensional symplectic over $\mathbb{R}$, with 1-dimensional kernel,

(c) 3 dimensional real orthogonal with signature $(1, 1, 1)$,

(d) 4-dimensional Hermitian with signature $(1, 3)$.

Problem 4 [6 points]

Let us consider vector spaces with norms again. Recall what a norm on a vector space is, e.g., in Kostrikin/Manin Chapter 10 ("Normed Linear Spaces"). Study this chapter carefully and prove the following very important theorem: All norms on finite-dimensional vector spaces are equivalent, i.e., for two norms $||\cdot||_1$ and $||\cdot||_2$ there are positive constants $c < c'$ such that $c||\ell||_1 \leq ||\ell||_2 \leq c'||\ell||_1$ for all ℓ in the vector space. (Note: The proof is also given in Chapter 10.7; I just want you to understand the proof and rephrase it in your own words.)